Electronics and Electricity

(draft under construction, poorly debugged)

I still do not have a clear understanding of what the state variables in a circuit are and why. For example, should it be the voltage and current of every point in the cirucit. Should it also include the voltage and current derivatives? Should it also include the voltage and current integrals, double derivatives ... Once we have this then we can decide what the state variables are for diodes, and for transistors .... It seems a clear understanding of this is lacking in just about every book I've bumped into.

It would also be useful to think of all the space that surrounds the "circuit" as being part of the circuit, only some parts are connected via zero resistance and some via zero admittance.

I dont understand teh whole issue with connecting circuits: Impedance In, Impedance Out, Impedance matching ...

Suppose a capcitor gets some current in. By doing so it acumulates charge so it seems like the current out should not be equal to the current in. Why is this point of view wrong?

Solving an electronic network means determining the currents or the voltages of each point of it.

1 Linear Components

- Resistor:

$$V(t) = RI(t) \tag{1}$$

- Capacitor:

$$V(t) = \frac{1}{C}Q(t) \tag{2}$$

  or equivalently

$$\frac{dV_c(t)}{dt} = \frac{1}{C}I(t) \tag{3}$$

- Inductor:

$$V(t) = L\frac{dI(t)}{dt} \tag{4}$$

- Linear Amplifiers:
- Operational Amplifiers:

where $V(t)$ represents the voltage drop accross the component, $I(t)$ the current accross teh component, and $Q(t)$ the electric charge.

It is best to think as every location in the circuit to have a mixture of resistance, capacitance and inductance. In practice we separatet these properties into components for which one of the three characteristics is dominant.

1.1 Linear System Analysis

Problem with this analysis is that it does not seem to take into consideration the initial conditions. I guess the issue is that the input the system is proved with is a sinusoid in the real line. There really is not initial condition in them. This analysis does not seem enough to handle transient effects.

1.2 The Bilateral Laplace Transform

Let $f : \Re^+ \to \mathcal{C}$. The Bilateral Laplace transform of f is a function $\hat{f} : \mathcal{C} \to \mathcal{C}$ defined as follows

$$\hat{f}(s) = \mathcal{L}[f(t)](s) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(t)e^{-st}dt \tag{5}$$

Example:
Let

$$u(t) = \begin{cases} 0, & \text{for } t < 0 \\ 1, & \text{else} \end{cases} \tag{6}$$

then

$$\hat{u}(s) = \int_0^\infty u(t)e^{-st}dt = \int_0^\infty e^{-st}dt = \left[\frac{-1}{s}e^{-st}\right]_0^\infty \tag{7}$$

$$= \frac{1}{s}, \text{ for } Re(s) > 0 \tag{8}$$

The subset of complex numbers s for which the Laplace transform integral exists is called the "Region of Convergence" or ROC. In the previous example, the region of convergence is the set of complex numbers with positive real part.

1.2.1 The Unilateral Laplace Transform

The unilateral tranform of $f(t)$ is the bilateral transform of $f(t)u(t)$. The unilateral transform is useful for analysis of linear systems in which the initial conditions are important.

The bilateral transform is useful for analysis of a system in which the effect of the initial conditions is negligeble, i.e., we can think of the input signal as having operated forever in the past. The main difference between the two transforms is in the differentiation property: An initial conditions terms appears in the unilateral transform but not in the bilateral transform.

1.2.2 Properties of the Unilateral Laplace Transform

1. **Linearity:**

$$\mathcal{L}[a_1 f_1(t) + a_2 f_2(t)](s) = a_1 \hat{f}_1(s) + a_2 \hat{f}_2(s) \tag{9}$$

2. **Differentiation:** Let

$$h(t) = \frac{df(t)}{dt} \tag{10}$$

Then

$$\hat{h}(s) = s\hat{f}(s) - f(0) \tag{11}$$

Proof: Using integration by parts we have

$$\int_0^\infty f'(t)e^{-st}dt = \left[f(t)e^{st}\right]_0^\infty + s\int_0^\infty f(t)e^{-st}dt \tag{12}$$

Note the dependency of the unilateral Laplace transform on initial conditions.

In the bilateral transform for signals with no beginning and end this dependency dissapears. *Show why*

3. **Multiple Differentiation** Using the differentiation rule twice we get that if

$$h(t) = \frac{d^2 f(t)}{dt^2} \tag{13}$$

then

$$\hat{h}(s) = s\hat{f}'(s) - f'(0) = s^2 \hat{f}(s) - sf'(0) - f(0) \tag{14}$$

In general if

$$h(t) = \frac{d^n f(t)}{dt^n} \tag{15}$$

then

$$\hat{h}(s) = s^n \hat{f}(s) - \sum_{k=0}^{n} s^k \frac{d^k f(0)}{dt^k} \tag{16}$$

4. **Integration:** Let

$$h(t) = \int_0^t f(u) du \tag{17}$$

Then

$$\hat{h}(s) = \frac{1}{s} f(s) \tag{18}$$

Proof:

$$f(t) = \frac{dF(t)}{dt} \tag{19}$$

Thus

$$f(t) = sF(s) - F(0) = sF(s) \tag{20}$$

5. **Complex Translation:** Let

$$\hat{h}(s) = \hat{f}(s+a) \tag{21}$$

Then

$$h(t) = e^{-at} f(t) \tag{22}$$

Proof:

$$\hat{h}(s) = \int_0^\infty f(t) e^{-(a+s)t} dt = \mathcal{L}[e^{-at} f(t)](s) \tag{23}$$

with the region of convergence for h switched accordingly.

6. **Complex Differentiation:** Let

$$\hat{h}(s) = \frac{d\hat{f}(s)}{ds} \tag{24}$$

Then

$$h(t) = -tf(t) \tag{25}$$

Proof:

$$\frac{d\hat{f}(s)}{ds} = \int_0^\infty f(t) \frac{d}{ds} e^{-st} dt = -\int_0^{-infty} tf(t) e^{-st} dt \tag{26}$$

with the region of convergence for h switched accordingly.

$f(t)$	$\hat{f}(s)$	ROC
$\delta(t)$	1	$\mathcal{C}$
$u(t)$	$\frac{1}{s}$	$\Re(s) > 0$
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\Re(s+a) > 0$
$u(t)\cos(\omega t)$	$\frac{s}{s^2+\omega}$	$\Re(s) > 0$
$u(t)\sin(\omega t)$	$\frac{\omega}{s^2+\omega}$	$\Re(s) > 0$

Table 1: Some Useful Laplace Transforms

7. **Time Scaling:** Let $a > 00$ and

$$h(t) = f(at) \tag{27}$$

Then

$$\hat{h}(s) = \frac{1}{a}\hat{f}(s/a) \tag{28}$$

Proof: Using change of variables $u = at$ we have

$$\hat{h}(s) = \int_0^\infty h(t)e^{-st}dt = \int_0^\infty f(at)e^{-st}dt = \frac{1}{a}\int_0^\infty f(a)e^{-\frac{s}{a}u}du \tag{29}$$

with the region of convergence for h scaled accordingly.

We have been ignoring the effect of the tranform on ROC. These effects should be considered part of the tranformation also

1.2.3 Inverse Laplace Transform

$$f(t) = \int_0^{\sigma+j\infty} \hat{f}(s)e^{st}ds \tag{30}$$

for any value of σ in the region of convergence.

1.2.4 Partial Fraction Expansion

It is the basic approach used to find inverse Laplace transforms of trasfer functions. A polynomical with real coefficients can be factored into a product of quadratics of the form $ax^2 + bx + c$, with $x \in \mathcal{C}$. Thus it can be factored into products of the form $(x - x_i)(x - x_j)$. The terms x_i, x_j are the roots of the polynomialos. These roots come in pairs. If $b^2 - 4ac > 0$ then x_i, x_j are real and distinct. If $b_2 - 4ac < 0$ then x_i, x_j are complex conjugates. If $b^2 - 4ac = 0$ then $x_i = x_j$.

Let

$$F(x) = \frac{N(x)}{D(x)} \tag{31}$$

where $N(x), D(x)$ are complex polynomials with real coefficients. Let $x_1, \cdots, x_n$ be the roots of $D(x)$. If all the roots are distinct (and therefore also real), then

$$F(x) = \sum_{i=1}^{n} \frac{k_i}{x - x_i} \tag{32}$$

To find k_i note

$$F(x)(x - x_i)|_{x=x_i} = \frac{N(x)}{\pi_{j\neq i}(x_i - x_j)} = k_i \tag{33}$$

Other methods exist to find the k_i factors when the roots are not distinct.

1.3 Application to Solving Differential Equations

Consider a system with input $X(t)$ and output $Y(t)$ characterized by the following differential equation:

$$X(t) = \sum_{k=0}^{m} b_k \frac{d^k Y(t)}{dt^k} \tag{34}$$

Taking the Laplace transform transform on both sides we have

$$\hat{X}(s) = \hat{Y}(s) \sum_{k=0}^{m} b_k\, s^k \left(1 - \frac{d^k Y(0)}{dY}\right) \tag{35}$$

Let the transfer function be defined as follows

$$H(s) \stackrel{\text{def}}{=} \frac{\hat{Y}(s)}{\hat{X}(s)} = \frac{\hat{X}(s)}{\sum_{k=0}^{m} b_k\, s^k \left(1 - \frac{d^k Y(0)}{dY}\right)} \tag{36}$$

and the impedance function be defined as follows

$$Z(s) \stackrel{\text{def}}{=} \frac{\hat{Y}(s)}{\hat{X}(s)} = \frac{\sum_{k=0}^{m} b_k\, s^k \left(1 - \frac{d^k Y(0)}{dY}\right)}{\hat{X}(s)} \tag{37}$$

2 Application to Analysis of Linear Circuits

2.1 Impedance Reactance, Admittance

Define the *"impedance"* Z of a circuit system as the transfer function when the initial conditions of the element are zero, the input is the potential difference applied to that element and the output is the current through the system

$$Z(s) = \frac{V(s)}{I(s)} \tag{38}$$

The real part of Z is called the *"resistance"*, the imaginary part is called the *"reactance"*.

The *"admittance"* Y of a circuit system is the the inverse of the impedance, i.e.

$$Y(s) = \frac{I(s)}{V(s)} \tag{39}$$

2.2 Impedance of Resistors, Capacitors and Inductors

Applying the Laplace transform to the resistor, capacitor and inductor equations we have

- **Resistors:**

$$V(t) = RI(t) \tag{40}$$

$$\hat{V}(s) = R\hat{I}(s) \tag{41}$$

Thus

$$Z_R = \frac{\hat{V}(s)}{\hat{I}(s)} \tag{42}$$

$$Y_R = \frac{\hat{I}(s)}{\hat{V}(s)} \tag{43}$$

- **Capacitors:**

$$\frac{dV(t)}{dt} = \frac{1}{C} I(t) \tag{44}$$

$$s\hat{V}(s) - V(0) = \frac{1}{C} \hat{I}(s) \tag{45}$$

Thus

$$Z_C = \frac{\hat{V}(s)}{\hat{I}(s)} = \frac{1}{sC} \tag{46}$$

$$Y_R = \frac{\hat{I}(s)}{\hat{V}(s)} = sC \tag{47}$$

- **Inductors:**

$$V(t) = L \frac{dI(t)}{dt} \tag{48}$$

$$V(s) = Ls\hat{I}(s) - I(0) \tag{49}$$

Thus

$$Z_L = \frac{\hat{V}(s)}{\hat{I}(s)} = sL \tag{50}$$

$$Y_R = \frac{\hat{I}(s)}{\hat{V}(s)} = \frac{1}{sL} \tag{51}$$

- **Linear Amplifiers:**
- **Operational Amplifiers:**

2.3 Connecting Circuits in Series and in Parallel

Using the conservation of laws we get that when two systems are connected in serial (Figure 1 Left)

$$V = V_1 + V_2 = Z_1 I + Z_2 I \overset{\text{def}}{=} IZ \tag{52}$$

Thus impedances add

$$Z = Z_1 + Z_2 \frac{1}{Y} = \frac{1}{\frac{1}{Y_1} + \frac{1}{Y_2}} \tag{53}$$

When the systems are connected in parallel (Figure 1 Right) we have

$$I = I_1 + I_2 = Y_1 V + Y_2 V \overset{\text{def}}{=} VY \tag{54}$$

Thus admittances add

$$Y = Y_1 + Y_2 \frac{1}{Z} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} \tag{55}$$

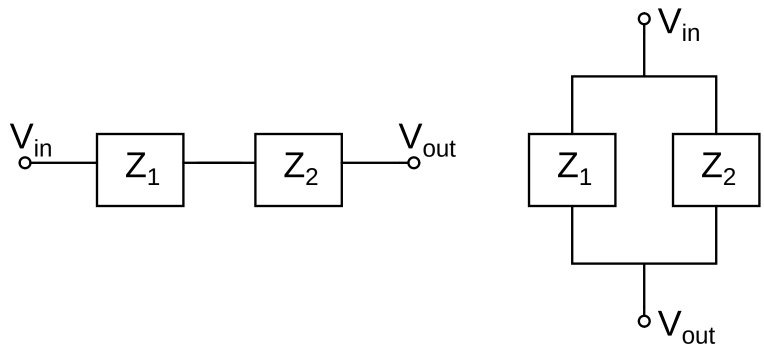

Figure 1: *Serial and Parallel Connection of Circuits*

2.3.1 RLC omponents in Series

$$Z(s) = Z_C(s) + Z_L(s) + Z_R(s) \tag{56}$$

In many applications the voltage sources produce sinusoids. Thus we are interested on the impedance to signals of the form $s = j\omega$, where

$$\omega \stackrel{\text{def}}{=} 2\pi f \tag{57}$$

is the angular frequency, measured in radians per second, and f is the frequency in Hertzs. In this case it is convenient to express Z as a function of ω

$$Z_R(\omega) = R \tag{58}$$

$$Z_C(\omega) = -\frac{j}{C\omega} \tag{59}$$

$$Z_L(\omega) j L\omega \tag{60}$$

where we used the fact that

$$\frac{1}{j} = \frac{j}{j^2} = -j \tag{61}$$

Note how capacitors and inductors can cancel each other's impedances, i.e.,

$$Z_C + Z_L = j(L\omega - \frac{1}{C\omega}) \tag{62}$$

Thus

$$\|Z_c + Z_L\| = L\omega^2 - \frac{1}{C^2\omega^2} \tag{63}$$

This is the reason why there are big cylinders (capacitors) outside industrial sites. The capacitors compensate for the inductance of the electrical machinery in the site.

2.3.2 Voltage Dividers

Consider the circuit in Figure 2. Note

$$I(t) = \frac{V_{in}(t)}{Z_1 + Z_2} \tag{64}$$

$$V_{out}(t) = Z_2 I(t) = \frac{Z_2}{Z_1 + Z_2} V_{in} \tag{65}$$

Voltage dividers are used to reduce an inpupt voltage to a smaller desired output voltage.

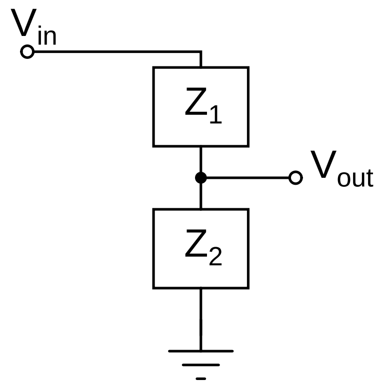

Figure 2: *Voltage Divider*

2.3.3 "Poor Boy" Current Sources

Ideal current sources provide constant current to a load independent of the load's impedance. The simplest way to approximate a voltage source is to connect a high impedance element Z in series with a high-voltage source V.

The current out of this circuit would be

$$I = \frac{V}{Z + Z_{Load}} \tag{66}$$

If V and Z are large in comparison with Z_{Load} then I is approximately independent of Z_{Load}.

Current sources can be approximated by having a large V_{in} and R.

2.3.4 Low Pass Filtering

Consider the circuit in Figure 3:Left. From (65) we have

$$V_{out} = \frac{-j\frac{1}{C\omega}}{R - j\frac{1}{C\omega}} V_{in} = \frac{1}{1 + jRC\omega} V_{in} \tag{67}$$

Thus

$$\|V_{out}\| = \frac{1}{\sqrt{1 + R^2C^2\omega^2}} V_{in} \tag{68}$$

2.4 Bode Plots

Bode plots describe the voltage gain in decibels (dB) as a function of frequency. The gain is decibels is defined as follows

$$dB = 10\log_{10}\frac{V_{out}^2}{V_{in}^2} = 20\log\frac{V_{out}}{V_{in}} \tag{69}$$

The **half-power frequency**, or **cutoff frequency**, or **3 dB breakpoint** is the frequency at which the energy out of the filter is half the energy in, i.e.

$$dB = 10\log_{10} 0.5 = -3.01dB \tag{70}$$

In the RC circiut above, we have that Since energy is proportional to V^2 we have

$$\|\frac{V_{out}}{V_{in}}\|^2 = \frac{1}{1+R^2C^2\omega^2} = \frac{1}{2} \tag{71}$$

$$\omega_{3dB} = \frac{1}{RC} \tag{72}$$

$$f_{3dB} = \frac{1}{2\pi RC} \tag{73}$$

Low-pass filters are sometimes approximated by two lines, one with the low frequency assymptote and one with the high frequency assymptote. For $\omega \leq\leq 1/RC$ the gain is approximated as 0 dB. For $\omega \geq\geq 1/RC$ the gain is approximately $-20\log_{10}\omega RC$. The two lines cross at $\omega = 1/RC$, which is for this reason also called the **corner frequency** or **break frequency**.

Also note for a gain of -20 dB the power has decreased by a factor of 100.

$$\|\frac{V_{out}}{V_{in}}\|^2 = \frac{1}{1+R^2C^2\omega^2} = \frac{1}{100} \tag{74}$$

$$\omega_{20dB} = \sqrt{99}\frac{1}{RC} \approx 10\frac{1}{RC} \tag{75}$$

$$f_{20dB} = 10\frac{1}{2\pi RC} \tag{76}$$

Figure 4:Left shows the frequency response curve of a filter wioth $R = 1000\Omega$ and $C = 0.1\mu F$. The 3dB breakpoint is 159.15 Hz.

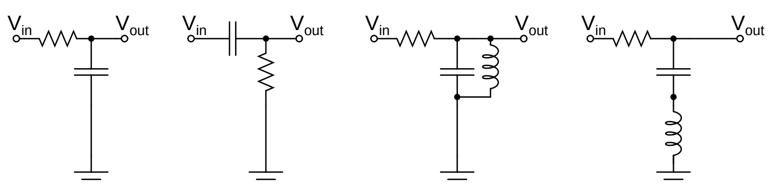

Figure 3: *From Left to Right: Low Pass Filter. High Pass Filter. Band Pass Filter. Notch Filter*

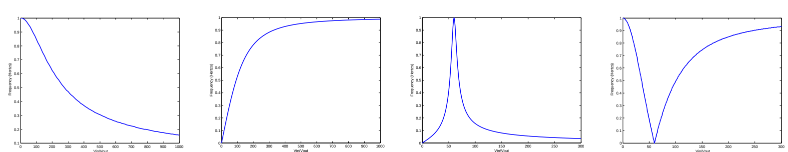

Figure 4: *From Left to Right: Frequency Response of a Low Pass Filter. High Pass Filter. Band Pass Filer. Notch Filter. The parameters of the Low Pass and High Pass Filters are:* $R = 1K\Omega$,$C = 0.1\mu F$. *The parameters fo the bBand Pass and Notch Filters are:* $R = 1K\Omega$, $C = 5mF$, $L = 1.41$ *Henrys*

Low Pass Filtering and Integration Note

$$I(t) = C\frac{dV(t)}{dt} = \frac{V_{in}(t) - V_{out}(t)}{R} \tag{77}$$

If we keep $V_{in} >> V_{out}$ then

$$C\frac{dV(t)}{dt} \approx \frac{V_{in}(t)}{R} \tag{78}$$

$$V(t) \approx \frac{1}{RC}\int_0^t V_{in}(s)ds \tag{79}$$

Note if $I(t)$ were constant, i.e., if we had a current source, then the integral would be exact.

2.4.1 High Pass Filtering

If we interchange the resistor and capacitor in a low pass filter, we get a high-pass filter. Consider the circuit in Figure 3:Center. From (65) we have

$$V_{out} = \frac{R}{R - j\frac{1}{C\omega}}V_{in} = \frac{1}{1 - j\frac{1}{RC\omega}}V_{in} \tag{80}$$

Thus

$$\|V_{out}\| = \frac{1}{\sqrt{1 + \frac{1}{R^2C^2\omega^2}}}V_{in} \tag{81}$$

The 3 dB breakpoint of the filter is as in the low pass filter

$$\|\frac{V_{out}}{V_{in}}\|^2 = 0.5 \tag{82}$$

$$\omega_{3dB} = \frac{1}{RC} \tag{83}$$

$$f_{3dB} = \frac{1}{2\pi RC} \tag{84}$$

Figure 4:Right shows the frequency response curve of a filter wioth $R = 1000\Omega$ and $C = 0.1\mu F$. The 3dB breakpoint is 159.15 Hz.

High Pass Filtering and Differentiation Note

$$I(t) = C\frac{d}{dt}(V_{in}(t) - V_{out}(t)) = \frac{V_{out}(t)}{R} \tag{85}$$

If we have R, C snakk so that

$$\frac{dV_{in}(t)}{dt} >> \frac{dV_{out}(t)}{dt} \tag{86}$$

then

$$V_{out} \approx R\frac{dV_{in}(t)}{dt} \tag{87}$$

2.4.2 Band Pass Filtering

Consider the circuit in Figure 3:Right. From (65) we have

$$V_{out} = \frac{\frac{1}{\frac{1}{Z_L}+\frac{1}{Z_C}}}{R + \frac{1}{\frac{1}{Z_L}+\frac{1}{Z_C}}}V_{in} = \frac{1}{1 + R\left(\frac{1}{Z_L} + \frac{1}{Z_C}\right)}V_{in} \tag{88}$$

$$= \frac{1}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)} \tag{89}$$

Thus

$$\|V_{out}\| = \frac{1}{\sqrt{1 + R^2\left(\omega C - \frac{1}{\omega L}\right)^2}}\|V_{in}\| \tag{90}$$

Thus the resonant (peak) frequency is at

$$\omega C = \frac{1}{\omega L} \tag{91}$$

$$\omega = \frac{1}{LC} \tag{92}$$

$$f = \frac{1}{2\pi LC} \tag{93}$$

Tog get the 3 dB breakpoints of the filter

$$\|\frac{V_{out}}{V_{in}}\|^2 = 0.5 \tag{94}$$

$$R^2\left(C\omega - \frac{1}{L\omega}\right)^2 = 1 \tag{95}$$

$$C\omega - \frac{1}{L\omega} = \pm\frac{1}{R} \tag{96}$$

$$C\omega^2 - \frac{1}{L} \pm \frac{\omega}{R} = 0 \tag{97}$$

$$\omega = \frac{\pm\frac{1}{R} \pm \sqrt{\frac{1}{R^2} + 4\frac{C}{L}}}{2C} \tag{98}$$

Note

$$\sqrt{\frac{1}{R^2} + 4\frac{C}{L}} > \frac{1}{R} \tag{99}$$

Thus our solution of interest is

$$\omega = \frac{\sqrt{\frac{1}{R^2} + 4\frac{C}{L}} \pm \frac{1}{R}}{2C} \tag{100}$$

and the 3dB bandwidth is

$$\Delta\omega = \frac{1}{RC} \tag{101}$$

$$\Delta f = \frac{1}{2\pi RC} \tag{102}$$

$$\tag{103}$$

Figure 4:Right shows the frequency response curve of a filter wioth $R = 1000\Omega$ and $C = 5\mu F$, $L = 1.401$ Henrys. The 3dB breakpoint is 159.15 Hz.

2.4.3 Notch Filtering

Consider the circuit in Figure 3:Right. From (65) we have

$$V_{out} = \frac{Z_L + Z_C}{Z_L + Z_C + Z_R} = \frac{1}{1 + \frac{Z_R}{Z_L + Z_C}} = \frac{1}{1 + \frac{R}{j\left(\omega L - \frac{1}{C\omega}\right)}} \tag{104}$$

Thus

$$\|V_{out}\| = \frac{1}{\sqrt{\left(1 + \frac{R}{\omega L - \frac{1}{C\omega}}\right)^2}}\|V_{in}\| \tag{105}$$

Thus the resonant (valley) frequency is at

$$\omega C = \frac{1}{\omega L} \tag{106}$$

$$\omega = \frac{1}{LC} \tag{107}$$

$$f = \frac{1}{2\pi LC} \tag{108}$$

To get the 3 dB breakpoints of the filter

$$\|\frac{V_{out}}{V_{in}}\|^2 = 0.5 \tag{109}$$

$$\left(1 + \frac{R}{\omega L - \frac{1}{C\omega}}\right)^2 = 2 \tag{110}$$

$$\frac{R}{\omega L - \frac{1}{C\omega}} = \pm 1 \tag{111}$$

$$\omega^2 L \pm \omega R - \frac{1}{C} = 0 \tag{112}$$

$$\omega = \frac{\pm R \pm \sqrt{R^2 + 4\frac{4L}{C}}}{2L} \tag{113}$$

Note

$$\sqrt{R^2 + 4\frac{L}{C}} > R \tag{114}$$

Thus our solution of interest is

$$\omega = \frac{\sqrt{R^2 + 4\frac{4L}{C}} \pm R}{2L} \tag{115}$$

and the 3dB bandwidth is

$$\Delta\omega = \frac{1}{RC} \tag{116}$$

$$\Delta f = \frac{1}{2\pi RC} \tag{117}$$

$$\tag{118}$$

Figure 4:Right shows the frequency response curve of a filter wioth $R = 1000\Omega$ and $C = 5\mu F$, $L = 1.401$ Henrys. The 3dB breakpoint is 159.15 Hz.

3 Analysis of Transient Responses

If we think of the previous sections as an application of the bilateral Laplace Transform (the Fourier transform if we force $s = 0 + j\omega$). This is useful for cases in which the signal has been operating for infinitely long thus erasing the effects of the initial conditions. In this section we focus on the analysis of transient effects

3.1 LR Circuit

Consider a resistor an inductor connected to a voltage source $V(t)$. We have

$$\frac{R}{L}I(t) + \frac{dI(t)}{dt} = \frac{1}{L}V(t) \tag{119}$$

with initial condition

$$I(0) = i_0 \tag{120}$$

$$\tag{121}$$

let the time constant$\tau = L/R$. (In RC circuits the time constant is RC). τ is the time necessary to decay 36.7 % of initial value. Thusxs

$$e^{\frac{1}{\tau}t}\frac{1}{\tau}I(t) + e^{\frac{1}{\tau}t}\frac{dI(t)}{dt} = \frac{d}{dt}e^{\frac{1}{\tau}t}I(t) = e^{\frac{1}{\tau}t}\frac{1}{L}V(t) \tag{122}$$

Integrating both sides

$$\int de^{\frac{1}{\tau}u}I(u)du = e^{\frac{1}{\tau}t}I(t) + C = \frac{1}{L}\int e^{\frac{1}{\tau}u}V(u)du \tag{123}$$

Thus

$$I(t) = -Ce^{-\frac{1}{\tau}t} + \frac{1}{L}e^{-\frac{1}{\tau}t}\int e^{\frac{1}{\tau}u}V(u)du \tag{124}$$

Where the constant C is chosen to satisfy the initial condition.

Note if there is no driving force, i.e., $V(u) = 0$ we have

$$I(t) = e^{-\frac{1}{\tau}t}C \tag{125}$$

This part of the solution is called the "Zero Input Response" or the "Transient Response", or the "Natural Response". Moreover, if the initial state is zero then

$$I(t) = \frac{1}{L}e^{-\frac{1}{\tau}t}\int_0^t e^{\frac{1}{\tau}u}V(u)du \tag{126}$$

This part of the solution is called the *" Zero State Response"*, or the "Force Response", or the *Steady State Response*.

Consider the case in which $V(t) = v$. We have

$$e^{\frac{1}{\tau}t}I(t) = -C\int e^{\frac{1}{\tau}u}vdu = \tilde{C} + \frac{v}{L\frac{1}{\tau}}e^{\frac{1}{\tau}t} = e^{-\frac{1}{\tau}t}\tilde{C} + \frac{v}{R} \tag{127}$$

Where

$$i_0 = 0 = \tilde{C} + \frac{v}{R} \tag{128}$$

Thus,

$$I(t) = \frac{v}{R}(1 - e^{\frac{1}{\tau}t}) \tag{129}$$

We can also solve the problem using the unilateral Laplace transform. Applying the transform to both sides of (139)

$$R\hat{I}(s) + sL\hat{I}(s) - I(0) = V(s) \tag{130}$$

Thus

$$\hat{I}(s) = \frac{\hat{V}(s) - I(0)}{R + sL} \tag{131}$$

Letting $I(0) = 0$ and $V(t) = vu(t)$ we get

$$\hat{I}(s) = \frac{v}{s(sL + R)} \tag{132}$$

First we find the weight associated with the root $s = 0$

$$\hat{I}(s) = \frac{k_0}{s} + \hat{G}(s) \tag{133}$$

Thus

$$k_0 = sI(s)|_{s=0} = \frac{v}{R} \tag{134}$$

$$\hat{G}(s) = I(s) - \frac{k_0}{s} = \frac{v}{s(sL+R)} - \frac{v}{Rs} = \frac{v}{s}\frac{R - sL - R}{R(sL+R)} \tag{135}$$

$$= -\frac{vL}{R}\frac{1}{sL+R} = -\frac{v}{R}\frac{1}{s+\frac{1}{\tau}} \tag{136}$$

Thus

$$\hat{I}(s) = \frac{v}{Rs} - \frac{v}{R}\frac{1}{s+\frac{1}{\tau}} \tag{137}$$

and taking the inverse Laplace in both sides

$$I(t) = \frac{v}{R}u(t) - \frac{v}{R}e^{-\frac{1}{\tau}t}u(t) = \frac{v}{R}(1 - e^{-\frac{1}{\tau}t})u(t) \tag{138}$$

3.2 RC Circuit

Consider a resistor and capacitor connected to a voltage source $V(t) = v$. We have

$$V(t) = \frac{Q(t)}{C} + I(t)R \tag{139}$$

with initial condition

$$Q(0) = q_0 \tag{140}$$

$$\tag{141}$$

Taking derivatives

$$\frac{dV(t)}{dt} = \frac{1}{C}I(t) + R\frac{dI(t)}{dt} \tag{142}$$

We will focus on the case of a constant voltage source, i.e.,

$$\frac{dI(t)}{dt} = -\frac{1}{\tau}I(t) \tag{143}$$

where the the time constant $\tau = RC$. τ is the time necessary to decay 36.7 % of initial value. Thus

$$I(t) = Ke^{-\frac{1}{\tau}t} \tag{144}$$

where K is determined by the initial conditions. Note

$$Q(t) = Q(0) + \int_0^t I(s)ds = q_0 + K\left(1 - \tau e^{-\frac{1}{\tau}t}\right) \tag{145}$$

We now $I(t) \to 0$ as $t \to \infty$ thus

$$V_c(\infty) \stackrel{\text{def}}{=} \lim_{t\to\infty} \frac{1}{C}Q(t) = v \tag{146}$$

Taking $\to \infty$ we get

$$V_c(\infty) = \frac{q_0}{C} + K = v \tag{147}$$

Thus

$$K = v - \frac{q_0}{c} = v - v_0 \tag{148}$$

where

$$v_0 \stackrel{\text{def}}{=} \frac{q_0}{C} \tag{149}$$

and we get

$$V(t) = v_0 + (v - v_o)\left(1 - \tau e^{-\frac{1}{\tau}t}\right) \tag{150}$$

3.2.1 Multiple Rs

If we have multiple Rs in series/parallel with a capactitor or an inductor, we use Thevenin's theorem to convert all the Rs into an equivalent R in series with the capacitor or resistor component.

Consider the following circuit: (Resistor1, Capacitor) in parallel with Resistor2 and in parallel with a voltage source V. In this case Resistor 2 has no effect whatsoever on the capacitor. However if there is a Resistor 2 and Capacitor are in parallel and both are in series with Resistor 1. In this case Resistor 1 and 2 have an effect on the capacitor. For example if Resistor2 = 0 Ohm then the assymptote voltage accross capacitor is V. However if Resistor 2 = Resistor 1 neq 0 then, since we have a voltage divider, the assymptote voltage accross capacitor is V/2

If there is a capacitor in parallel with a resistor and connected to a voltage source -¿ The resistor

3.3 Discrete Time Computer Simulations

Consider a LRC circuit in series, characterized by the differential equation

$$V(t) = \frac{1}{C}Q(t) + L\frac{dI(t)}{dt} + RI(t) \tag{151}$$

with initial conditions

$$Q(0) = Q_0 \tag{152}$$
$$I(0) = I_0 \tag{153}$$

We can approximate the solution using Euler's method, i.e. we choose a small step size Δ_t and iterating

$$\Delta I(t) = \frac{1}{L}\left(V(t) - \frac{1}{C}Q(t) - RI(t)\right)\Delta_t \tag{154}$$
$$I(t + \Delta_t) = I(t) + \Delta I(t) \tag{155}$$
$$Q(t + \Delta_t) = Q(t) + I(t)\Delta_t \tag{156}$$

Note a conductor is a circuit with no impedance, i.e., $R \to 0, L \to 0, C \to \infty$. Thus to make a CR circuit, we let $L \to 0$. In such case as $L \to 0$

$$\frac{\Delta I(t)}{\Delta_t} \to \infty \tag{157}$$

This expresses the fact that in such condition $I(t)$ catches up instantaneously with its desired value i.e.

$$V(t) = \frac{1}{C}Q(t) + R\frac{dQ(t)}{dt} \tag{158}$$

or equivalently

$$\frac{dV_c(t)}{dt} = \frac{V(t) - V_c(t)}{RC} \tag{159}$$

where

$$V_c(t) \stackrel{\text{def}}{=} \frac{Q(t)}{C} \tag{160}$$

Thus, using Euler's method

$$V_c(t + \Delta_t) = V_c(t) + \frac{V(t) - V_c(t)}{RC} \Delta_t \tag{161}$$

with initial condition

$$V_c(0) = \tilde{v}_0 = \frac{q_0}{C} \tag{162}$$

To get an LR circuit we let $C \to \infty$, i.e., $\frac{1}{C} = 0$.

4 Input Output View

This is a crucial issue. Note circuits seem to be inherently non-feedforward. Every component has an effect on every other component. On the other the feed-forward view of processing is very important. We want a signal to drive other components and we do not want the other components to have an effect on our signal. Designing sources with low impedance and loads with high impedance seems to be the way this is achieved

4.1 Thevening Equivalent

It seems to me, but would like to confirm on this, that a 2 terminal component of a linear circuit is fully characterized by its transfer function, i.e., a function that tells how it converts voltage into current and viceversa.

By definition, in any linear circuit the current (and voltage) at any point is a linear combination of the independent voltage sources V_i and independent current sources I_j in the circuit

$$I(z) = \sum a_i(z) V_i(z) + \sum b_j(z) I_(z) \tag{163}$$

where I is the current at a fixed point in the circuit and z is a complex number.

Suppose we now have an entire linear circuit with two terminals to which we can plug other circuits. To characterize the behavior of the entire circuit all we need is the transfer function of the circuit, i.e., how it converts voltage into current or viceversa. To do so we apply an external voltage to the two external terminals of the circuit and observe the current flowing throught the terminals. By linearity we have that

$$I(z) = \sum_i a_i(z) V_i^{int}(z) + \sum_j b_j(z) I_j^{int}(z) - c(z) V^{ext}(z) \tag{164}$$

where *int* stands for independent voltage and current sources internal to the circuit. Now let

$$V_{Thev}(z) \stackrel{\text{def}}{=} c(z) \left(\sum_i a_i V_i^{int}(z) + \sum_j b_j I_j^{int}(z) \right) \tag{165}$$

$$R_{Thev}(z) \stackrel{\text{def}}{=} \frac{1}{c(z)} \tag{166}$$

Thus

$$I(z) = \frac{V_{Thev}(z) - V^{ext}}{R_{Thev}} \tag{167}$$

Which corresponds to the transfer function of a voltage source $V_{Thev}(z)$ in series with an impedance R_{Thev}.

To estimate $V_{Thev}(z)$ apply an external voltage $kV^{ext}(z)$ and find the value of k for which current does not flow accross the external terminals. This thus corresponds to the z component of the voltage that could be measured with the terminals in open circuit.

To estimate R_{Thev} set allo internal sources to zero and measure the impedance accross the external terminals: Since the internal sources are set to zero we get

$$I(z) = \frac{-V^{ext}(z)}{R_{Thev}(z)} \tag{168}$$

and

$$R_{Thev}(z) = \frac{-V_{ext}(z)}{I(z)} \tag{169}$$

The Thevening equivalence is very important for circuit compositionality, i.e., we can connect two entire circuit and analyze the behavior of the two coupled circuits. The original circuit with two external terminals (e.g., an amplifier) is called the source. The circuit we attaching to the orignal (e.g., a speaker) is called the load. The load is also called the "input" circuit and the source the "output" circuit. This later designation takes the point of view of the source (it outputs signals that serve as input to the load circuit). I personally find this confusing for i tend to think of the source as providing input to the load.

Using the Thenevin equivalence we see that the source and load circuits have a voltage divider relationship. Thus

$$V_{Load} = V_{Source} \frac{Z_{Load}}{Z_{Load} + Z_{Source}} \tag{170}$$

In many cases we want the source voltage signal to be as unaffected as possible by the load circuit. Particularly damaging is if the impedance of the load varies with the signal source levels, we want the effect of that impedance to be as small as possible otherwise the signal will be distorted. Thus we want $Z_{Load} >> Z_{Source}$. A good rule of thumb is Z_{Load} at least 10 times bigger than Z_{Source}.

The source circuit is said to "drive" the load. Good source circuits have very small impedance, in the order of milli Ohms. Because they do not "bend" under load they are called "stiff" sources.

Measuring instruments, on the other hand, ought to have little influence on the measured circuit, thus they are designed to have very high impedance.

5 Cable Model

Definitions Figure 5 shows a standard cable model. The different components are defined as follows:

- Rdc - Commonly referred to DCR which is the series resistance of a cable at zero frequency.

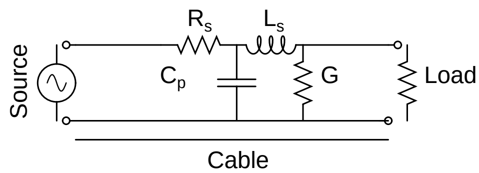

Figure 5: *Cable Model*

Figure 6: *Left: RCA; 2 Center: BNC; Right: F-connectors*

- Rac - The resistive portion of the cables series resistance as a function of frequency due to skin effect [1].
- Rs - Total Series Resistance (mohms) measured tip to tip at one end of the cable while the other end is shorted. Note: Rs = Rac + Rdc (minus instrumentation inaccuracies identified below)
- Ls - Series Inductance (uH) measured tip to tip at one end of the cable while the other end is shorted. Check out our article on Cable Inductance for more info.
- Cp - Parallel Capacitance (pF) measured tip to tip at one end of the cable while the other end is open circuited.
- G - G conductance (1 / Rdielectric). The major relevancy roles of the dielectric in this application are to serve as isolation between the two conductors and control the capacitance of the cable based on the conductor spacing and dielectric constant.

5.1 Cable and connector types

source: audioholics.com

- Zip cable : Two cords side by side
- Coax cable: Shield outside runs ground, Line core inside The composition of the shield in will determine how effective it is at preventing interference; the best shields for broad-frequency coverage, all the way from low-frequency hum to RFI, are combination shields consisting of both a heavy braid (wire woven around the dielectric in a sort of mesh) and a full-coverage foil (usually aluminum, wrapped around the dielectric). The most common way

[1] It is the tendency of alternating current to flow near the surface of a conductor, thereby restricting the current to a small part of the total cross-sectional area and increasing the resistance to the flow of current. The skin effect is caused by the self-inductance of the conductor, which causes an increase in the inductive reactance at high frequencies, thus forcing the carriers, i.e., electrons, toward the surface of the conductor. At high frequencies, the circumference is the preferred criterion for predicting resistance than is the cross-sectional area.

to save a bit of money on cable construction here is in the braid; lower-quality coaxial cables will sometimes use an aluminum braid (significantly less conductive, and consequently less effective, than copper), with poor coverage–as low as 60%. Generally, one should look for a copper braid, with 95% coverage (100%, while it'd be nice, isn't possible because the nature of wire braid is that it has holes caused by the way it's woven), coupled either with a full-coverage layer of foil or with another heavy braid.

- RCA (Radio Corporation America) Plug and Jack: the most common connector type on consumer gear for composite and component video, as well as for both digital and analog audio. It's not a very good connector, as connectors go, but as it's what equipment manufacturers have given us, it's what we often have to use. RCA jacks color-coded yellow on a device usually are composite video inputs or outputs.
- BNC is the standard connector for most video signals on professional gear, and is showing up increasingly on high-end consumer gear as well. It will be labeled similarly to the RCA, indicating composite video (one connection), Y/C s-video (two connections), Y/Pb/Pr (three connections), or one form or another of RGB. The most common confusion with BNCs, in our experience, is that people often assume the female connector is a male; the problem is that both the male and female connectors have what looks like a pin in the center. On closer inspection, however, you'll see that a female BNC's "pin" is actually a receptacle for the male pin. A panel-mounted BNC will ALWAYS be female; a cable-mounted BNC will almost always be male, though there are exceptions (such as our breakout adapters, which have female BNCs to join with standard cable-mount male BNCs).
- F-connector: is the screw-on type connection used for most antenna and cable TV connections. F-connectors are rarely used for anything other than RF; the one notable exception being that they were used as digital audio connectors on some laser disk players.

5.2 Audio Cable: Source audioholics.com

My Polk home stereo speakers are set for 8 Ohms, and a recommended amplification of 20 to 150 Watts. Since $W = VI = RI^2$ the recommended current is 1.58 to 4.33 Amps. Wow, these are huge currrents!

Because speakers are driven at low impedance (typically 4 or 8 ohms) and high current, speaker cables are, for all practical purposes, immune from interference from EMI or RFI, so shielding isn't required. The low impedance of the circuit also tips the balance of concern from capacitance, which is important in interconnect use, to inductance, which, while a concern, can be controlled only to a limited degree. The biggest issue in speaker cables, from the point of view of sound quality, is simply conductivity; the lower the resistance of the cable, the lower the contribution of the speaker cable's resistance to the damping factor, and the flatter the frequency response will be. While one can spend thousands of dollars on exotic speaker cable, in the end analysis, it's the sheer conductivity of the cable, and (barring a really odd design, which may introduce various undesirable effects) little else that matters.

The load is the resistance (impedance) presented by the speakers that is seen by the amplifier. This can also include any crossovers and circuits connected to the speakers. When the load decreases, the amplifier's output increases. There is less resistance to the current, and the speakers can draw more power from the amp. Drawing more power than the amplifier was designed for will damage the amp. Every amplifier is designed to handle a certain load. For home amplifiers this

Frequency	L_s	R_s	C_p
(Hz)	(μ H)	(m Ω)	(pF)
100	0.194	2.20	20
1K	0.195	2.19	18.7
100K	0.184	6.90	15.6
1M	0.172	48.5	13. 6

Table 2: Characteristics of high quality Audio Cable (Brand AV, 10AWG). Source: audioholics.com

number usually starts with 8 ohms. With car amplifiers it is usually 4 ohms. All amplifiers can handle a higher resistance (load), but they will produce less output. Most quality amplifiers can also handle a lower resistance. Most car amps can handle a 2 ohm load, while some can go as low as 1/2 ohms. (source lalena.com)

Do a spice analysis with source of 1V at different frequenceies and a load of 4 ohms for speaker system)

The typical 12AWG zip (twin parallel cords) cord has about 3.4 mohms of loop resistance per foot, .200uH/ft of Inductance and about 20pF/ft of capacitance.

If we examine the data from our various Speaker Cable Face Off articles, particularly Speaker Cable Face Off I, we see that a 10ft length of 12AWG zip cord terminated into a 4 ohm load only experiences -.088dB of loss at 20kHz and about 2nsec of group delay. Increase the cable length to about 50ft and we do see losses surmount to about -.745dB and 209nsec. Note that at 20 kHz, a phase shift of 36 degrees represents 5 microseconds (almost 24 times larger than our 50ft cable delay), this delay being considered as close to the limit of human directionality perception.

Further examining the data from our article Skin Effect Relevance in Speaker Cables we illustrated that model for human hearing is highly insensitive to ultra high frequency response and also discussed that music above 8kHz is harmonic in nature with minimal content at the high frequency extremes. It is a good idea however to use lower gauge wire (10 AWG or less) for runs greater than 50ft to minimize these losses especially when driving loudspeakers with a low impedance (4 ohms or less) profile. Note however a 5 microsecond delay has only been detected with any degree of certainty, under controlled laboratory conditions, within the approximate 3500Hz region where the ear is most sensitive. Again this detection must be in an extremely quiet environment, certainly below NC20, almost anechoic in fact. Source (http://www.audioholics.com/techtips/audioprinciples/interconnects/SpeakerCablelength.php, April 26, 2006).

5.3 Video Cable

NTSC signals range 1MHz to 5 MHz.

Characteristic Impedance Consider the circuit in Figure **??** representing a cable infinitely long. As we close the switch electrons will propagate at close to the speed of light prgresively chargint the capacitors as they go along. A current will run through the voltage source. The impedance created by this line is called the "characteristic impedance" and it can be shown to be

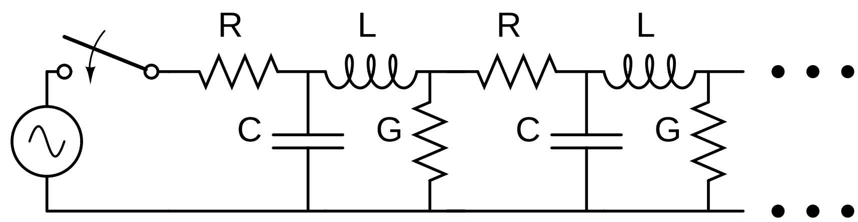

Figure 7: *An infinitely long transmission line.*

$$Z(f) = \sqrt{\frac{R + j2\pi f L}{G + j2\pi C}} \tag{171}$$

It is unclear to me why the equation above is not a function of time. It may be that it is just a fact or it may be that this assumes the switch has been closed for infintely long. I dont know which one is true. It is interesting to observe how the characteristic impedance behaves as a function of the frequency f

$$Z(f) \approx \begin{cases} \sqrt{\frac{R}{G}}, & \text{for } f = 0 \\ \sqrt{\frac{R}{j2\pi f C}}, & \text{for audio signal } (f \text{ order 1KHz}) \\ \sqrt{\frac{R}{j2\pi f C}} & \\ \sqrt{\frac{L}{C}}, & \text{for electronic video signal } (f \text{ order 1MHz}) \end{cases} \tag{172}$$

Since characteristic impedance is important only at high frequencies, the number reported is the limit as $f \rightarrow infty$, which is $\sqrt{L/C}$.

If a cable were many times longer than the signals' wavelenght then it could be considered to be of infinite lenght. Note the wavelenght (speed of light/ frequency) of a 3 Khz signal is about 30 Km and the wavelenght of a 3Mhz signals is about 30 meters. Thus the lenght of typical audio and video cables cannot be considered infinitely long.

I dont understant the following statement. Why is the case that when the cable is short with respect to the wavelenght, characteristic impedanced is not an issue?

Most wires will have a speed of travel for AC current of 60 to 70 percent of the speed of light, or about 195 million meters per second. An audio frequency of 20,000 Hz has a wavelength of 9,750 meters, so a cable would have to be four or five *kilometers* long before it even began to have an effect on an audio frequency. That's why the characteristic impedance of audio interconnect cables is not something most of us have anything to worry about. Normal video signal rarely exceed 10 MHz. That's about 20 meters for a wavelength. Those frequencies are getting close to being high enough for the characteristic impedance to be a factor. High resolution computer video signals and fast digita signals easily exceed 100 MHz so the proper impedance matching is needed even in shor cable runs.

If the cable is terminated by a load of impedance different from the characteristic impedance, the traveling waves of voltage and current will be reflected. The voltage ratio between the incident wave and the reflected wave can be whown to be

$$\rho = \frac{V_i}{V_r} = \frac{Z_l - Z_c}{Z_l + Z_c} \tag{173}$$

Impedance	75 Ohms
Inductance:	0.115 microhenries/feet
Capadicantce:	20.5 pf/Feet
Propagaation Velocity:	66 % (speed of light?)
Delay:	1.54 ns/feet
Conductor DC resistance:	49 Ohms/1000 feet
Shield dC resistance:	2.6 Ohms/1000 feet
Attenuation at 1 Mhz	0.6 dB/100 feet
Attenuation at 10 Mhz	1.1 dB/100 feet
Attenuation at 100 Mhz	3.4 dB/100 feet
Attenuation at 1000 Mhz	12 dB/100 feet

Table 3: Typical numbers for video cable (Belden 8264)

Table 4: tab:videocablespecs. Note $\sqrt{L/C} = 74.898\Omega$

where Z_c is the characteristic impedanced of the cable and Z_l the impedance of the load. Note

$$\rho = \begin{cases} 0, & \text{if } Z_l = Z_c\text{, i.e, impedance matching case} \\ -1, & \text{if } Z_l = 0\text{, i.e., closed circuit case} \\ 1, & \text{as } Z \to \infty\text{, i.e., open circuit case} \end{cases} \tag{174}$$

Reflections are bad because:; (1) we lose power of the original signal; (2) We create reflection noise. Thus the ideal case is when the characteristic impedance of the cable equals the characteristic impedance of the load. Note reflections will also happen at any point if the cable does not have uniform impedance. It is quite remarkable that the impedance of the cable, is independent of the cable's lenght (I know this is true for the case with impedance matched load and cable. I dont know whether it is also the case for non-matched impedances).

Video cable is manufacture to have a characteristic impedance of 75 Ohms. Video devices also have impedances of 75 Ohms, to maximize power transfer and minimize reflections.

6 Junction Diode Model

Use the specs of the 1n4001 as the standard. It is currently the most popular rectifying diode

6.1 Notation

- $\Re^+$: Real numbers.
- $\Re^+$: Nonnegative real numbers.
- $\mathcal{C}$: Complex numbers.
- Forward biased: Current flows from anode (positive end) to cathode (negative end).
- Reverse biased: Current flows from cathode to anode.
- Leakage current: Same as reverse current. Current when diode is reverse biased.

- Forward current: Current when diode is forward biased.
- Avalanche current: Huge current drop at the breakdown voltage of a rerverse biased diode.

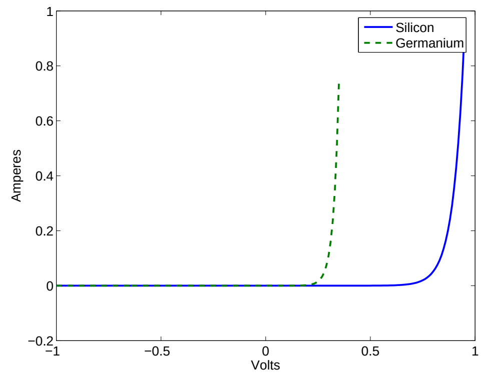

Figure 8: *The I-V curve for Silicon and Germanium diodes*

The diode model will use the following parameters:

- q: Electron charge: 1.610^{-19} Coulumbs.
- k: Boltzmann's constant[2]: 1.3810^{-23}.
- T: Temperature in Kelvin (Centigrades + 273.16).
- V_T: Thermal Voltage.

$$V_T \stackrel{\text{def}}{=} \frac{kT}{q} \tag{175}$$

  At standar room temperature is 27 degrees centigrades we have

$$V_T \approx 0.026 \tag{176}$$

- η: Emision coefficient. $\eta = 2$ for Silicon junctionss, and $\eta = 1$ for Germanium juncions. I've also seen $\eta = 1.4$ for Silicon.
- I_s: Saturation reverse currrent. Current when the diode is reverse biased. At 27 degrees centigrades it is about 10^{-6} A for Germanium and about 10^{-8} A for Silicon. It approximately doubles every 10 degrees C rise in temperature.Other figures I've seen are 10^{-4} for Germanium and 10^{-10} for Silicon.
- I_{max} Maximum forward current. For LEDs, $I_f(max)$ is about 30 mA. For diodes is in the order of 1 A. The popular 1N4001 diodes have an $I_{max} = 1$ A.
- V_b: Peak inverse voltage, or reverse breakdown voltage, beyond which the diode breaks down. In range of -0.5V for LEDs and -50 V for rectifiying diodes. The popular 1N4001 diodes have an $I_{f(max)} = -50$ V.

[2]This constant derives its name from the Austrian physicist Ludwig Boltzmann (1844-1906). It represents the increment in the energy of a molecule per Kelvin degree. It is approximately $1.3807x10 - 23$ Joules per Kelvin degree.

- Knee (or threshold) voltage: The point at which the diode suddenly begins to conduct, i.e., the forward current suddenly increases. For silicon diodes a standard accepted value is 0.7Volts,and 0.3 for Germanium diodes, 1.1 for Gallium Arsenide diodes, 2V for LEDs. Some say it corresponds to a a standard current "threshold" of 1mA.

A good first approximation model for the range of operation of the diode, both in forward and reverse bias, is given by the following equation: for $I < I_{max}$ and $V > V_b$.

$$I = I_s(e^{\frac{V}{\eta V_T}} - 1) \tag{177}$$

or equivalently

$$V = \eta V_T \log\left(\frac{I + I_s}{I_s}\right) \tag{178}$$

Where V is the voltage difference accross the diodes, and I the current accross the diode. This is known as the Schottky diode equation, or simply the diode equation.

Note for $I >> I_s$

$$V \approx \eta V_T \log\left(\frac{I}{I_s}\right) \tag{179}$$

The value of V at maximum forward current thus depends on the emission coefficient η. Rectifying diodes have $\eta \approx 1$ and LED $\eta \approx 12$, thus the voltage drop in LED is about twice as much as the drop in rectifying diodes.

6.2 Typical Values

- Typical silicon rectifying diode: 1n4001: $\eta = 2.57, I_s = 1.25 \times 10^{-6} A$
- Typical germanium diode:
- Typical LED: $\eta 12.59, I_s = 4.45 \times 10^{-5}$.

6.3 Parameter estimation

Given two points of the iv curve η and I_s can be estimated as follows

$$\eta = \frac{V_2 - V_1}{v_T \log(i_2/i_1)} \tag{180}$$

$$I_s = \frac{i_1}{e^{v1/(\eta v_T)}} \tag{181}$$

6.4 LED

Light emittind diodes have a typical forward voltage drop of 1.5 to 3 volts instead of 0.7 volts of rectifying diodes. Their typical operating current is about 5 mA to xs20 mA. When using an LED with a voltage source, a resistor in series must be added to limit current to be about 20 mA. Also, LED have much lower PIV rating than rectifying diodes. Rectifying dioes in the order of 50 Volts, and LEDs in the order of 5 Volts.

6.5 Simplified Models

- Ideal Diode: Open circuit (i.e., infinite resistance) for $V < 0.7$ and close circuit (i.e., zero resistance) for $V \geq 0.7$.

- Locally linear model:

  I approximated the diode equation for the following parameters: $V_T = 0.026, \eta = 1.4, I_s = 10^{-10}, PIV = -10V, I_{max} = 0.5A$. I used linear regression local to 3 regions: negative voltage, 0 to threshold, and above threshold. For a threshold of 0.7 I get teh following locally linear approximation:

$$I = \begin{cases} V/(66671 \times 10^6) & \text{for } -10 < V < 0 \\ V/192 & \text{for } 0 < V < 0.7 \\ (V - 0.7)/0.32 + 0.7/192 & \text{for } V > 0.6, I < 0.5 \end{cases} \tag{182}$$

  For a threshold of 0.6 we get the following locally linear approximation

$$I = \begin{cases} V/(66671 \times 10^6) & \text{for } -10 < V < 0 \\ V/2250 & \text{for } 0 < V < 0.6 \\ (V - 0.6)/0.97 + 0.6/2250 & \text{for } V > 0.6, I < 0.5 \end{cases} \tag{183}$$

  This suggests modeling the diode in 3 regions. When reverse biased it behaves as a resistor in the order of 10^5 Mega Ohms. When forward biased with voltage less than 0.6, it behaves as a resistor in the order of 2K Ohms. When forward biased with voltage more than 0.6 it behaves like a 1 Ohm resistor in series with a voltage source of $-0.6/0.97 + 0.6/2250 \approx 0.6$ $Volts$

6.6 Diode-Resistor circuit

Consider the circuit in Figure 9 left: a 5 volt source connected in series to a 220 Ohms resistor and a standard rectifying diode. We get two unknowns: V, the voltage difference accross the diode junction, and I, the circuits, current. We also get two constraints:

$$5 = V + 200I \tag{184}$$

$$I = 10^{-12}(e^{\frac{V}{0.026}} - 1) \tag{185}$$

Thus

$$\frac{5 - V}{200} = 10^{-12}(e^{\frac{V}{0.026}} - 1) \tag{186}$$

The right side of Figure 9 displays the problem graphically. The continuous line represents the diode equation, and the dotted like the resistor equation. The solution is at the intersection point, at which point both equations are satisfied.

One way to find the solution iteratively is as follows: (1) Start with an estimate of V. Given that V plug it on the diode equation to get an estimate of I. (2) Plug the I into the resistor equation to get a refined estimate of V. (3) Iterate.

An approximate solution can be obtained by assuming an idealized diode with constant voltage drop of 0.6 independent of current. Then the current would be

$$(5 - 0.6)/220 = 0.02 \text{ Amps} \tag{187}$$

The power disipated by the resistor is $0.02 \times 4.4 = 0.088$ Watts. Standard resistors can safely dissipate about 100 mWatts.

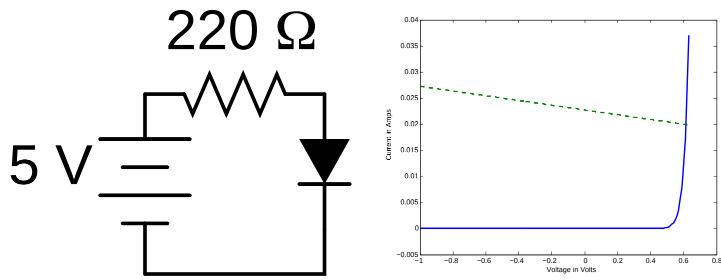

Figure 9: *Simple Diode-Resistor Circuit*

6.7 Diode-Resistor-Capacitor circuit

$$V(t) = RI(t) + V_c(t) + h(I_t) \tag{188}$$

where

$$V_c(t) \stackrel{\text{def}}{=} \frac{Q(t)}{C} \tag{189}$$

Given a small step size Δ_t and ignoring high order terms

$$V(t) = RI(t) + V_c(t - \Delta_t) + I(t)\Delta_t + h(I(t)) \tag{190}$$

Where V_c is the voltage across the capacitor and h is the diode equation. We can think of this as an Resitor-Diode problem

$$V(t) - V_c(t - \Delta_t) = I(t)(R + \Delta_t) + h(I(t)) \tag{191}$$

If we know $V_c(t - \Delta_t)$ We can solve for $I(t)$ as in the Resistor-Diode problem an then update to $V_c(t)$

$$V_c(t) = V_c(t - \Delta_t) + \Delta_t I(t) \tag{192}$$

Note we are using the constraint that

$$I(t) = \lim_{\Delta_t \to 0} \frac{Q(t) - Q(t - \Delta t)}{\Delta t} \tag{193}$$

It is also instructive to use the following alternative method. Assuming we have a good estimate of $I(t)$ and Δ_t is small we can linearize h as follows

$$V(t + \Delta_t) = I(t + \Delta_t)R + V_c(t + \Delta_t) + h(I(t)) + [I(t + \Delta_t) - I(t)]h'(I(t)) \tag{194}$$

where

$$h'(I) \stackrel{\text{def}}{=} \frac{dV}{dI} = \frac{\eta V_T}{I + I_s} \tag{195}$$

is known as the diode's "dynamic impedance". We can think of this as a circuit with a current dependent resistor

$$R + h'(I(t)) \tag{196}$$

a capacitor C, and a current dependent voltage source

$$V(t) - h(I(t)) + I(t)h'(I(t)) \tag{197}$$

Thus

$$V_c(t+\Delta_t) = V_c(t) + \frac{1}{C\hat{R}(t)}\big(\hat{V}(t+\Delta_t) - V_c(t)\big)\Delta_t \tag{198}$$

$$I(t+\Delta_t) = C\frac{V_c(t+\Delta_t) - V_c(t)}{\Delta_t} \tag{199}$$

where

$$\hat{R}(t+\Delta_t) \stackrel{\text{def}}{=} R + h'(I(t)) \tag{200}$$

$$\hat{V}(t+\Delta_t) \stackrel{\text{def}}{=} V(t+\Delta_t) + I(t)h'(I(t)) - h(I(t)) \tag{201}$$

$$\tag{202}$$

Note this linearization depends on having a good estimate of $I(t)$, this is particularly important for the initial conditions. Given initial condition $V_c(0)$ and $V(0)$ we can solve $I(0)$ by thinking of it as a diode-resistor problem. Hereafter we can continue solving iteratively using the linearize h approach. Figure 10 shows a simulation using this approach, of a 10 microFarad capacitor charging through a 200 Ohm resistor in series with a diode and a 3Volt power supply. The left side of the Figure shows the voltage accross the capacitor, the right side the dynamic impedance of the diode.

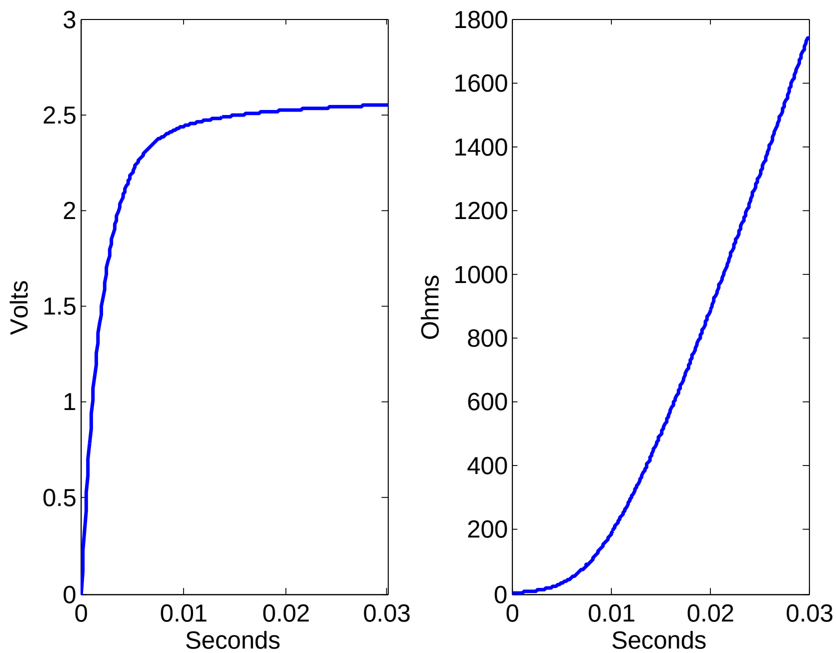

Figure 10: *Left: A capacitor charging through a resistor in series with a diode; Right: Dynamic impedance of the diode as the capacitor charges.*

7 Junction Transistor

7.1 Conventions: Junction Transistor: Ebers-Moll Model

Idealized parameters for 2N3904

Parameter	Typical	Range
h_{FE}	200	[30,300]
I_b	10 mA	[0.1, 100] mA
V_{CE}^{sat}	0.07V	< 0.2V
V_{BE}^{sat}	0.7V	< 0.95V

Based on this we will use the following convention for a typical idealized transistor: $\beta = h_{FE} = 100$, $V_{CE}^{sat} = 0.2V$, $V_{BE}^{sst} = 0.7V$.

Use the specs 2N3904 as the standard NPN and the 2N3906 as the standard PNP transistor. They are the most popular in the US Here are some paopular transistors: PN2222A, 2N3904, 2N4401, 2N2222A, 2N2222, 2N3906, PN2907, 2N4403 , KSP10, 2SC2570, 2SC2570A, Nowadays, the most visibly popular transistor, at least in the USA, is the 2N3904, a small NPN component that can be found in hundreds, if not thousands, of types of electronic devices. The 2N3906 (its PNP counterpart) and the 2N2222 (NPN) are also very popular. Radio Shack, seem to be favoring the 2N3904 over the 2N2222 as their generic NPN switching transistor The NTE123AP is equivalent to the 2N2222 (It is the third most popular device in NTE. The first most popular is the NTE125 rectifier, and the second most popular is the heat sink compound. The NTE125 is equal. to a 1N4007 A popular rectifier is the 1N4001 Designations: 2N... means American transistors, BC... are European transistors. American designation: 1N.... is used for semiconductors with 1 junction like a diode i.e. 1N4148, 2N.... is used or semiconductors with 2 junctions like a transistor i.e. 2N2222 3N... for other semiconductors like smal IC like optocopler i.e. 3N435, Take the 2N2222 as a standard and use its parameters as typical parameter values.*

Figure 11 Display the junction transistor model proposed by Ebers and Moll (EM) in 1954 [**?**]. This model is also called the coupled diode model. The model is adequate for static, DC behavior. Gummel and Poon [**?**] proposed a more advanced model that subsummes Ebers-Moll and deals with transient behavior.

Transistors have 4 operating states (See Table 5): (1) Active forward, or linear forward; (2) Active reverse, or linear reverse; (3) Saturation, or "ON" ; (4) Cutoff, or "OFF".

	BC Forward Biased	BC Reverse Biased
BE Forward Biased	Saturated	Forward Active
BE Reverse Biased	Forward Active	Cut-off

Table 5: Modes of transistor operation. In addition breakdown modes occur if V_{CE} or V_{BE} are too extreme.

The following model covers the 4 modes of operation.

7.2 Model Parameters

There are two types of parameters: Those describing the limits of normal opertion of the transistor and those describing the behavior of the transistor during normal operation. Regarding the second type the model has only three free parameters:

- I_{ES}: Base-emitter transport saturation current. Typical valule: 50 nA.
- I_{CS}: Base-collector transport saturation current. Typicdal value: 64 nA
- α_F : Forward transport factor. Typical value: 0.96.

The parameter α_R, reverse transport factor can be obtained from the following relationship:

$$I_{ES}\alpha_F = I_{CS}\alpha_R \tag{203}$$

From the typical values of the other parameters, we get a typical value for α_R of 0.75.

7.3 Model Equations

The junctions in transistor are built to have emission coefficient $\eta = 1$. Thus from the diode equation we get (177)

$$I_F = I_{ES}\left(e^{\frac{V_{BE}}{v_T}} - 1\right) \tag{204}$$

$$I_R = I_{CS}\left(e^{\frac{V_{BC}}{v_T}} - 1\right) \tag{205}$$

$$\tag{206}$$

where the thermal voltage v_T is defined in (**??**). In addition, since current conserves (see current sign conventions in Figure 11),

$$I_E = I_F - \alpha_R I_R \tag{207}$$

$$I_C = -I_R + \alpha_F I_F \tag{208}$$

$$I_E = I_B + I_C \tag{209}$$

Thus

$$I_B = (1-\alpha_F)I_F + (1-\alpha_R)I_R \tag{210}$$

Since voltage conserves we get

$$V_{CE} = V_{CB} + V_{BE} = V_{BE} - V_{BC} \tag{211}$$

Note in this model for the base current to be negative we need I_F or I_R to be negative and add up to a negative value. Note the current is lower bounded as follows

$$I_B \geq -\big((1-\alpha_F)I_{ES} + (1-\alpha_R)I_{CS}\big) \tag{212}$$

7.3.1 Useful Relationships

- (I_F, I_B) as a function of (I_C, I_B)

$$I_F = \frac{I_B + (1-\alpha_R)I_C}{1-\alpha_F\alpha_R} \tag{213}$$

$$I_F = \frac{\alpha_F I_B - (1-\alpha_F)I_C}{1-\alpha_F\alpha_R} \tag{214}$$

- I_C as a function of (I_B, V_{CE})

$$I_C = G(V_{CE})I_B + H \tag{215}$$

where

$$G(V_{CE}) = \frac{\alpha_F I_{ES}\, e^{V_{CE}/V_T} - I_{CS}}{(1-\alpha_R)I_{CS} + (1-\alpha_F)I_{ES}\, e^{V_{CE}/V_T}} \tag{216}$$

$$H = \beta\big((1-\alpha_F)I_{ES} + (1-\alpha_R)I_{CS}\big) + I_{CS} - \alpha_F I_{ES} \approx 0 \tag{217}$$

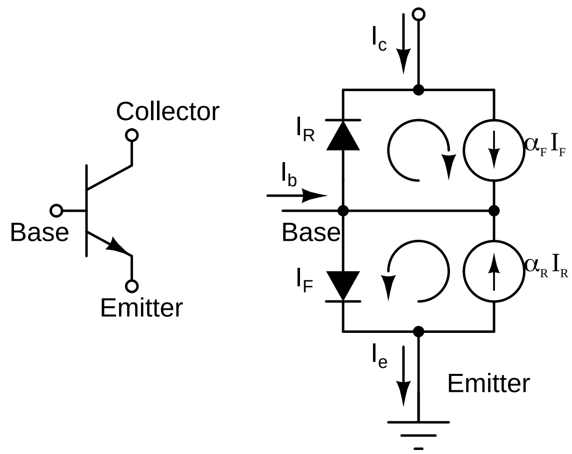

Figure 11: *Ebers-Moll Junction Transistor model: Two diodes with 2 current sources coupled to the diodes. The small arrows outside the circuits display the current direction signs used in this document.*

We treat H as beign effectively 0, in which case

$$G(V_{CE}) \approx \frac{I_C}{I_B} \tag{218}$$

Note

$$\lim_{VCE \to \infty} G(V_{CE}) = \lim_{VCE \to \infty} \frac{I_C}{I_B} = \frac{\alpha_F}{1-\alpha_F} = \beta \tag{219}$$

which is independent of I_B I have noticed however in Spice simulations that the ratio I_C/I_B with large V_{CE} increases with I_B quite significantly.
The following approximation is useful

$$G \approx \frac{\alpha_F I_{ES}\, e^{V_{CE}/V_T}}{(1-\alpha_R) I_{CS} + (1-\alpha_F) I_{ES}\, e^{V_{CE}/V_T}} = \beta \,\text{logistic}\left(\frac{V_{CE}}{VT} - \theta\right) \tag{220}$$

where

$$\beta \stackrel{\text{def}}{=} \frac{\alpha_F}{1-\alpha_F} \tag{221}$$

$$\theta \stackrel{\text{def}}{=} \log\left(\frac{1-\alpha_R}{1-\alpha_F}\frac{I_{CS}}{I_{ES}}\right) \tag{222}$$

Figure **??** Left shows the current gain G as a function of V_{CE} and the logistic approximation.

$$\frac{dG}{dV_{CE}} \approx \frac{G\,(1-G/\beta)}{VT}; \tag{223}$$

Figure **??** Right shows the derivative of G with respect to V_{CE} and the derivative of the logistic approximation.

- V_{CE} as a function of I_B, I_C

  Inverting (216) is difficult. Instead we will invert the logistic approximation (220) as follows

$$V_{CE} = V_T\left(\theta - \log\left(\frac{I_B}{I_C}\,\beta - 1\right)\right), \;\text{ for } I_C < \beta\, I_B \tag{224}$$

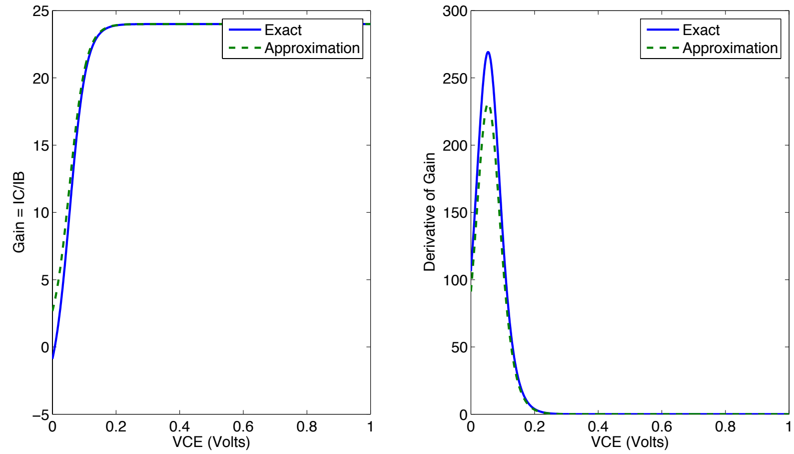

Figure 12: **Left:** *Transistor's Current Gain* (I_C/I_B) *as a function of* V_{CE}. **Right:** Derivative of Gain with respect to V_{CE}. Each figure shows the exact gain function and the logistic approximation.

$$\frac{dV_{CE}}{dI_C} = \frac{1}{I_B}\left(\frac{V_T}{\frac{I_C}{I_B}\left(1-\frac{I_C}{I_B}\frac{1}{\beta}\right)}\right), \quad \text{for } I_C < \beta\, I_B \tag{225}$$

- V_{BC} as a function of (I_B, V_{CE})

$$V_{BC} = V_T \log\left(\frac{I_B + (1-\alpha_R)I_{CS} + (1-\alpha_F)I_{ES}}{(1-\alpha_R)I_{CS} + (1-\alpha_F)I_{ES}\, e^{\frac{V_{CE}}{V_T}}}\right) \tag{226}$$

Thus the value of V_{CE} for which $V_{BC} = 0$ is

$$\tau = V_T \log\frac{I_B + (1-\alpha_F)I_{ES}}{(1-\alpha_F)I_{ES}} \tag{227}$$

If $V_{CE} > \tau$ then $V_{BE} < 0$. If $V_{CE} < \tau$ then $V_{BE} > 0$

7.4 State Analysis

Transistors have 3 terminals and thus 6 variables of interest: $I_B, I_C, I_E, V_{BE}, V_{CB}, V_{CE}$. Of these 2 are redundant due to conservation of current and current

$$I_E = I_B + I_C \tag{228}$$

$$V_{CE} = V_{CB} + V_{BE} = V_{BE} - V_{BC} \tag{229}$$

This leaves us with 4 independent variables. Now note if V_{BE}, V_{BC} are known then from (204) and (**??**) all the current variables follow. Equations can also be used to derive the 6 state variables from any 2 of the set. Thus, the state of the transistor is fully determined by 2 state variables.

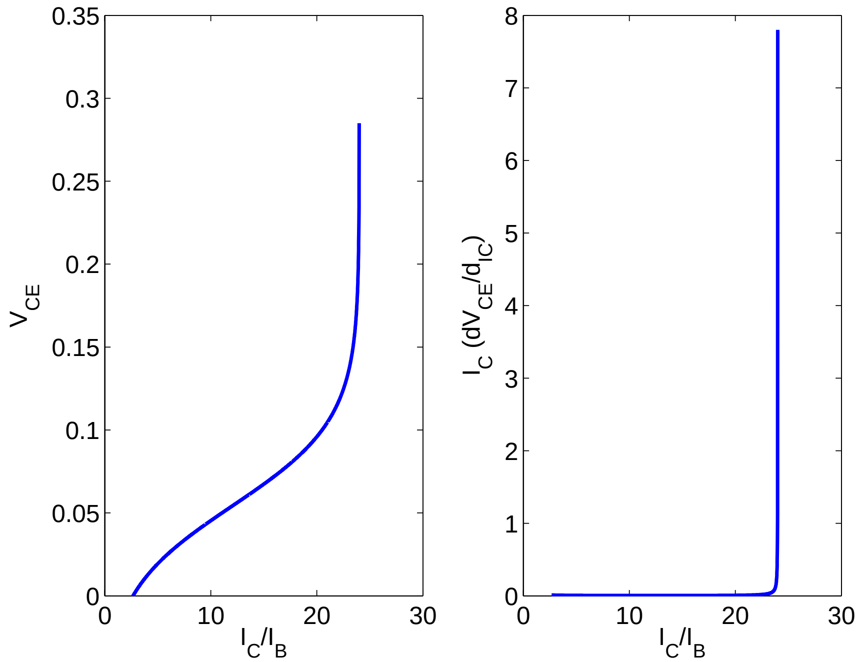

Figure 13: **Left:** *Approximation of* V_{CE} *as a function of* I_C/I_B. **Right:** I_B *times the derivative of* V_{CE} *with respect to* I_C*, based on logistic approximation.*

7.5 Simplified Model for Forward Active Case

In active forward state $I_R \approx 0$, in which case the EM model can be simplified as displayed in Figure 15. From $I_R = 0$ it follows that

$$I_E = I_F - \alpha_R = I_F \quad (230)$$

$$I_C = \alpha_F I_F = \alpha_F I_E \quad (231)$$

$$I_B = I_E - I_C = (1 - \alpha_F) I_E \quad (232)$$

Note[3]

$$\beta \stackrel{\text{def}}{=} h_{FE} \stackrel{\text{def}}{=} \frac{I_C}{I_B} = \frac{\alpha_F}{1 - \alpha_F} \quad (233)$$

Thus if I_B is known the other two currents are determined:

$$I_C = \beta I_B \quad (234)$$

$$I_E = I_B + I_C = I_B(1 + \beta); \quad (235)$$

Moreover, due to the diode equation, V_{BE} is also determined:

$$V_{BE} = v_T \left(1 + \log\left(\frac{I_b}{(1 - \alpha_F) I_{ES}}\right)\right) \quad (236)$$

Finally, if in addition to I_B, V_{CE} is also known then V_{BC} is determined:

$$V_{BC} = V_{BE} - V_{CE} \quad (237)$$

[3]This provides a standard way to estimate α_F. A similar procedure can be used to estimate α_R while operating in active reverse mode.

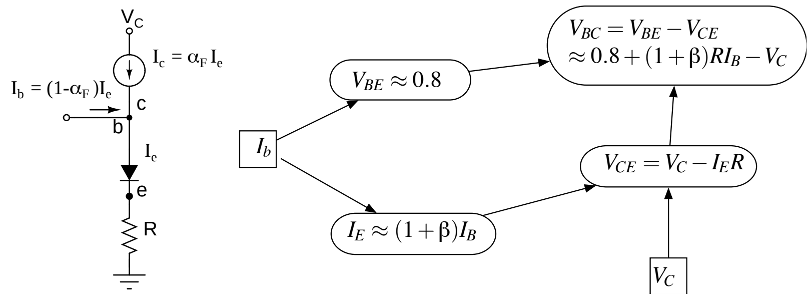

Figure 14: *Left: Simplified Model for Forward Active case. Right: Representation of variable dependencies. I_b is a current source and V_c a voltage source.*

Further simplification of the active forward model can be achieved by modeling the diode as an ideal diode with constant voltage drop ≈ 0.7 (See Right side of Figure 15). The diode model can be futher simplified as a constant voltage drop (i.e., voltage source (See Figure **??**).

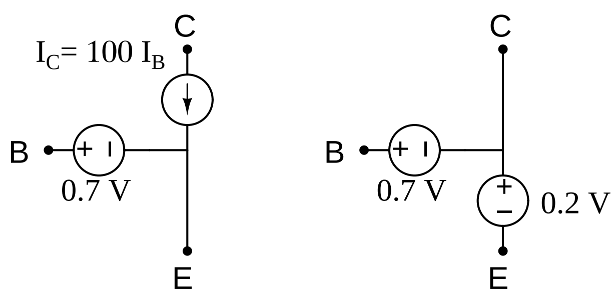

Figure 15: *Left: Simplified Model for a Typical Transistor's Forward Active Mode. Right: Simplified Model for a Typical Transistor's Saturation Mode.*
Representation of variable

7.5.1 Example Circuit 1

Figure 16 Left shows the simplest transistor circuit I could think of. It makes the point that you can control to transistor variables independently and the rest is set. The independent variables in this circuit are I_B and V_C. From conservation of voltage we have

$$V_{BC} = V_T \log\left(\frac{I_B + (1-\alpha_F)I_{CS} + (1-\alpha_R)I_{ES}}{(1-\alpha_R)I_{CS} + (1-\alpha_F)I_{ES}\, e^{\frac{V_C}{V_T}}}\right) \quad (238)$$

$$V_{BE} = V_C + V_{BC} \quad (239)$$

$$I_C = \alpha_F I_F - I_R \quad (240)$$

$$I_E = I_C + I_B \quad (241)$$

Figure 17 Right shows the 4 dependent variables of the transistor for the 2 given independent variables. The base current were: 0 mA, 0.025 mA, 0.050 mA, 0.075 mA, 0.1 mA.

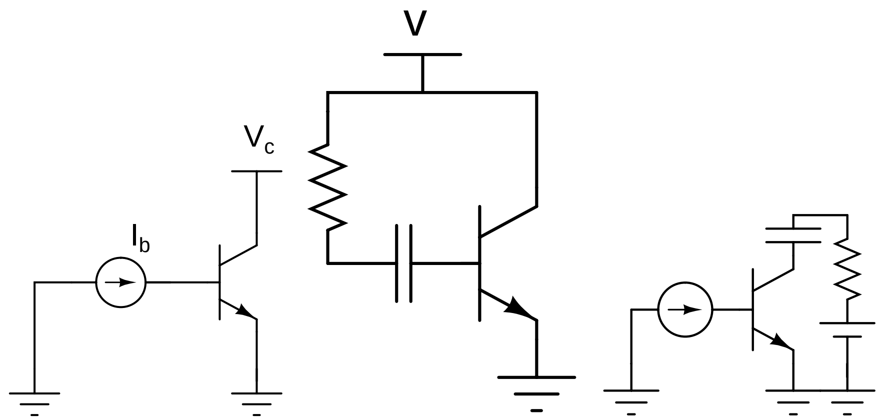

Figure 16: *Left: Simple Transistor Circuit. Right: Four transistor dependent variables as a function of V_{CE}, I_B. Right: Resistor, Capactior, Transistor Circuit*

7.5.2 Example Circuit 2: Base Resistor, Capacitor

Consider the circuit in Figure 16 Right. To getn intuitive understanding we will use a discrete-time Euler approximation. Consider a time t for which V_{CA}, the voltage accross the capacitor, is known

$$v = I_B(t)R + V_{CA}(t) + V_{BE}(t) \tag{242}$$

$$V_{BE} = h(I_B(t)) = v + V_T \log\left(\frac{I_B(t) + (1-\alpha_F)I_{CS} + (1-\alpha_R)I_{ES}}{(1-\alpha_F)I_{CS} + (1-\alpha_R)I_{ES}\, e^{\frac{V_C}{V_T}}}\right) \tag{243}$$

We can find $I_B(t)$ by assuming a value of $V_{BE}(t)$ and solving for $I_B(t)$ in (242), then using $I_b(t)$ to solve for $V_{BE}(t)$ in (243), and iterating until convergence. Once $I_B(t)$ is known we can get the voltage accross the capacitor for the next time step

$$V_{CA}(t+\Delta_t) = V_{CA}(t) + I_B(t)\Delta_t \tag{244}$$

Successive application of this technique will converge to the solution as $\Delta_t \to 0$.

If we have a good estimate for $I_B(t)$ and Δ_t is small, we can linearize h

$$v = I(t+\Delta_t)R + V_{CA}(t+\Delta_t) + h(I(t)) + [I(t+\Delta_t) - I(t)]h'(I(t)) \tag{245}$$

where

$$h'(I) \stackrel{\text{def}}{=} \frac{dV_{BE}}{dI} = \frac{V_T}{I_B(t) + (1-\alpha_F)I_{CS} + (1-\alpha_R)I_{ES}} \tag{246}$$

We can think of this as a circuit with a current dependent resistor

$$R + h'(I(t)) \tag{247}$$

a capacitor C, and a current dependent voltage source

$$V(t) - h(I(t)) + I(t)h'(I(t)) \tag{248}$$

Thus

$$V_c(t+\Delta_t) = V_c(t) + \frac{1}{C\hat{R}(t)}\big(\hat{V}(t+\Delta_t) - V_c(t)\big)\Delta_t \tag{249}$$

$$I(t+\Delta_t) = C\frac{V_c(t+\Delta_t) - V_c(t)}{\Delta_t} \tag{250}$$

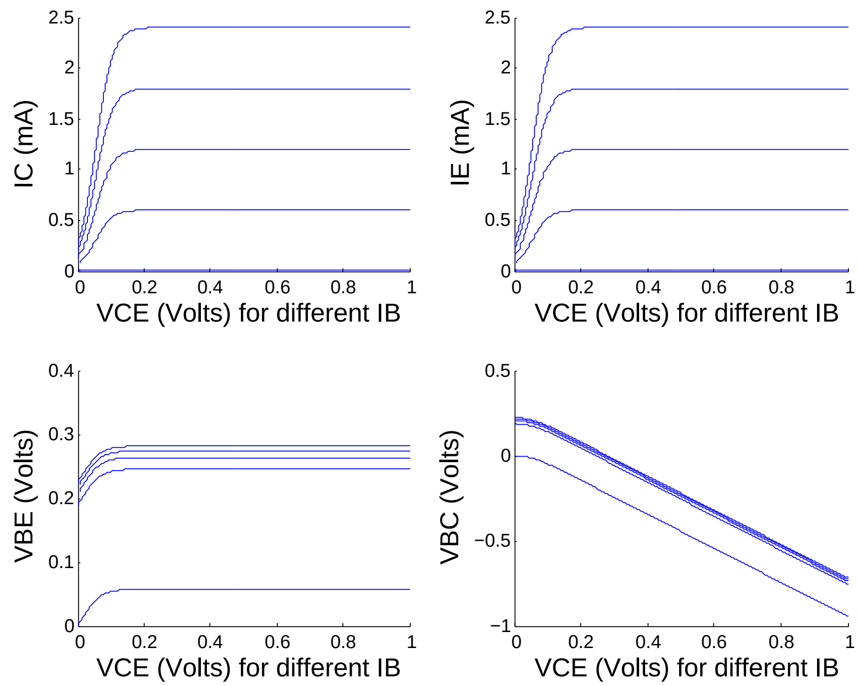

Figure 17: *Four transistor dependent variables as a function of V_{CE}, I_B. The base current were: 0 mA, 0.025 mA, 0.050 mA, 0.075 mA, 0.1 mA.*

where

$$\hat{R}(t+\Delta_t) \stackrel{\text{def}}{=} R + h'(I(t)) \tag{251}$$

$$\hat{V}(t+\Delta_t) \stackrel{\text{def}}{=} V(t+\Delta_t) + I(t)h'(I(t)) - h(I(t)) \tag{252}$$

(253)

Note this linearization depends on having a good estimate of $I(t)$, this is particularly important for the initial conditions. Given initial condition $V_c(0)$ and $V(0)$ we can solve $I(0)$ by thinking of it as a diode-resistor problem. Hereafter we can continue solving iteratively using the linearize h approach.

Figure **??** shows a simulation using this approach, of a 10 microFarad capacitor charging through a 2000 Ohm resistor in series with a diode and a 3Volt power supply. The left side of the Figure shows the voltage accross the capacitor, the right side the dynamic impedance of the diode.

7.5.3 Example Circuit 3: Collector Resistor, Capacitor

Consider circuit on Figure 16 Right.

$$v = RI_C(t) + V_q(t) + V_{CE}(t) \tag{254}$$

$$I_C(t) = f(I_B, V_{CE}) \tag{255}$$

where V_q is the voltage accross the capacitor. For initial condition $t = 0$ we are given $V_q(0)$. We can solve for $I_C(0), V_{CE}(0)$ by assuming a value for $I_C(0)$ solving

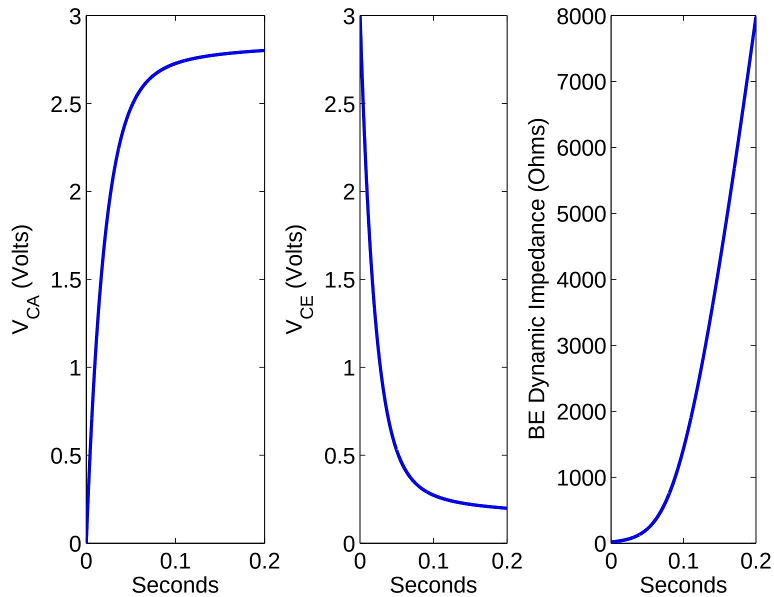

Figure 18: *Euler approximation of Resistor, Capacitor, Transistor Circuit. Resistor was 2K Ohms, and Capacitor 10 μ Farads*

for $V_{CE}(0)$ on (254), using the obtained value to get $I_C(0)$ on (255) and iterating until convergence. For any t, if $I_C(t), V_q(t)$ are known we can aproximate $V_q(t+\Delta_t)$

$$V_q(t+\Delta_t) = V_q(t) + \Delta_t \frac{I_C(t)}{C} \tag{256}$$

which provide the initial condition of the capacitor for the next time step.

I tried the linearization approach by using the logistic approximation of the mapping from V_{CE} to I_C given I_B. This did not work well I'm not sure why. Also for the approach in which we iterate back and forth between the two equations, I had to do it with a small change rate, ie., instead of moving to the I_C or V_{CE} that solves an equation for the other variable fixed, we just moved in the direction of the solution using an exponential smoothing with $\gamma = 0.99$, otherwise the approach would not converge Figure 19 are the results of a simulation of this circuit with $V = 1Volt, C = 0.1\mu F$, $R = 160Ohms$, $I_B = 0.1mA$, $\beta = 24$.

7.5.4 Common Port Configurations

Figure 20 shows the three typical input-output configurations: (1) Common Base; (2) Common Collector and (3) Common Emitter.

- **Common Base:** Input is the collector currect and the collector/base voltage. Output is the emitter current and the base/emitter voltage.
- **Common Collector:** Input is the base current and the collector/base voltage. Output is the emitter current and the base/emitter voltage.
- **Common Emitter:** Input is the base current and the collector/emitter voltage. Output is the collector current and the base/collector voltage.

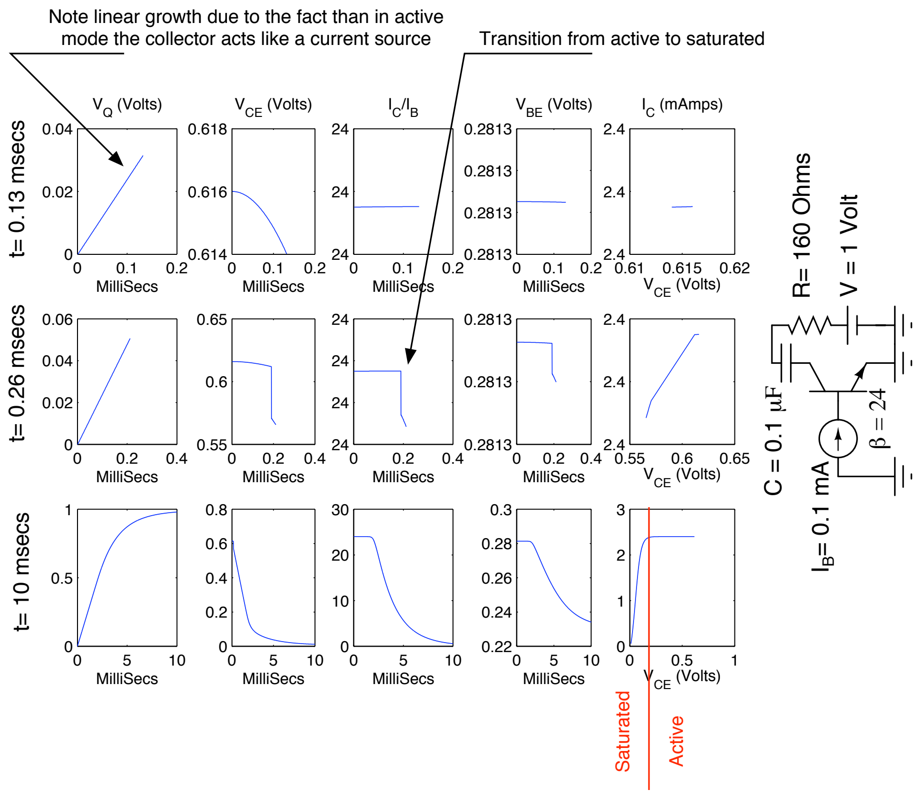

Figure 19: *Simulation of resitor and capacitor in series with collector. The base is current clamped.* $V = 1Volt$, $C = 0.1\mu F$, $R = 160Ohms$, $I_B = 0.1mA$, $\beta = 24$. *The title of each column indicates the units for the Y axis. The rows represent snapshots at different points in time.*

Calling things input-output is a bit misleading since the output does have an effect on the input. I still need to clarify in what sense it is ok to call things input-output. I suspect it has to do with high outuput inpedance and low input impedance.

7.5.5 Properties of Active Forward State

- For a fixed I_B the current accross C is independent of the voltage applied between C and E, i.e, the collector behaves as a current source.
- For an operating transistor (forward active, and saturation) $V_{BE} = 0.6$, due to the rectifying diode from base to emitter. Consider the circuit in Figure 21. We have

$$V_B = 0.6 + I_E R_E \tag{257}$$

$$I_E = \frac{V_B - 0.6}{R_E} \tag{258}$$

Thus, as $R_E \to 0$ then $I_E \to \infty$ so for this circuit it is important to have $R_E > 0$.

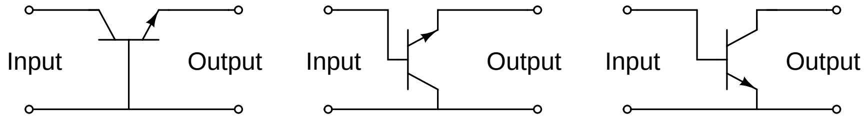

Figure 20: *Three standard configurations: Left: Common Base; Center: Common Collector; Right: Common Emitter*

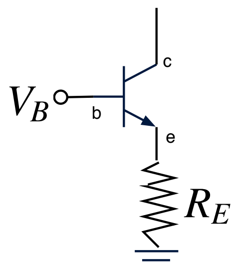

Figure 21: *Example Transistor Circuit*

7.6 Simplified Model for Saturation State

Defined by the fact that both junctions are forward biased and teh current flows from the collector through the emitter.

The right side of Figure 15 describes the variable dependencies in the active forward model. For a fixed V_C as I_b increases V_{BC} will increase. When V_{BC} becomes large enough to forward bias the BC junction the system goes into saturated mode. The base and collector currents at that point will be approximately

$$I_B^{sat} \approx \frac{V_{BC}^{threshold} + V_C - 0.8}{(1+\beta)R} \tag{259}$$

$$I_C^{sat} \approx \beta I_B^{sat} \approx \frac{V_{BC}^{threshold} + V_C - 0.8}{R} \tag{260}$$

When V_{CE} reaches the saturation value V_{CE}^{sat} the base collector junction becomes forward bias and I_C cannot increase any longer. Typically V_{CE}^{sat} is about 0.05 to 0.2 Volts. We say the transistor is in "saturation" or 'ON state". Thus active forward state is characterized by $V_{CE} >> V_{CE}^{sat}$. The saturation state is characterized by $V_{CE} = V_{CE}^{sat}$.

In this state I_C is constant and independent of I_B, V_{CE} stays constant, ie.e., $V_{CE} = V_{CE}^{sat}$, independent of I_B, and V_{BE} stays

A common saturation model uses 3 constant current sources with $V_{BE} = 0.8$ Volts and $V_{CE} = 0.2$ Volts. Thus $V_{CB} = V_{CE} - V_{BE} = 0.2 - 0.8 = -0.6V = V_C - V_B$. Thus $V_B = V_C + 0.6V$. Note at saturation $V_{BC} = 0.6 > V_\gamma \approx 0.5$ and thus the BC diode is actually forward biased.

Saturation can also be defined by $V_{BC} \geq V_{BE}$. The saturation voltage can be found analytical by finding the point for which $V_{BE} = V_{CE}$

7.7 Cutoff Mode

The "cutoff", or "OFF" mode, is determined by the fact that both junctions are revered biased.

At this point there is a very small collector leackage current, but it is so small that it can be ignored. Both te base-emitter and base-collector are reverse biased.

7.8 Reverse Active

The amplificationfactor is usually less than the forward-active mode.

7.9 Breakdown

When V_{CE} pases the breakdown voltage the forward-biased base-collector goes into breakdown mode and I_c increases rapidily. Something akin happens when V_{CE} is too negative.

7.10 Collector Emitter Characteristic

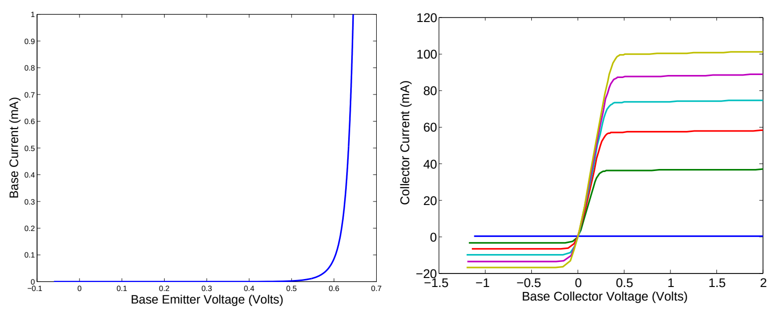

Figure 22: **Left:** *Input characteristic curve for Common Emitter configuration:It displays Base Current as a function of Base-Emitter Voltage. The Collector Emitter voltage was fixed to 0.1.* **Right:** *Collector Emitter Characteristics for the 2N3904 (Based on Spice Model). Curves are for following base currents in mA: (0, 0.4, 0.8, 1.2, 1.6,2)*

Is a collection of curves each ploting I_C as a function of V_{CE} for different values of I_B, for the base-emitter junction forward biased. Three regions can be observed: Forward Active, Saturation and Breakdown.

- Forward Active: In this region I_C changes little as a function of V_{CE}, it behaves as a current source. In the 2N2222 it is the region between 0.3 V to 50V.
- Break-Down: I_C inreases rapidely as a function of V_{CE} due to the avalanche effect. In the 2N2222 Break-Down starts for $V_{CE} > 50V$.
- Saturation: The remaining region. It is characterized by the fact that the collector current increases as a function of V_{CE}. In 2N2222 covers V_{CE} between 0 and 0.3 Volts.

7.10.1 Example Circuit

Consider the circuit in Figure 23. As we decrease the resistance R_b the current I_b will increase. While the transistor operates in the linear (active) region, then the collector current I_c will increase proportionally to I_b, i.e., $I_c = \beta I_b$. As I_c increases V_{CE} decreases due to the increase in voltage drop accross the resistor R_c:

$$10 = V_{ce} + I_c R_c \tag{261}$$

constant, typically at 0.6 Volts.

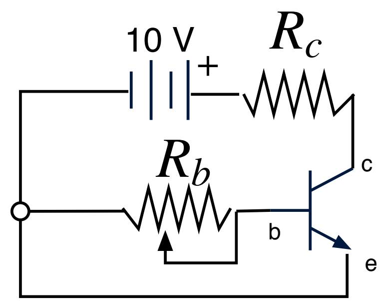

Figure 23: *Example Transistor Circuit*

7.11 Example Circuit

Consider the circuit in Figure 24 where the two diodes and linked current sources model an NPN transistor. First we put the diode equations:

$$I_F = I_{ES}\left(e^{\frac{V_{BE}}{v_T}} - 1\right) \tag{262}$$

$$I_R = I_{CS}\left(e^{\frac{V_{BC}}{v_T}} - 1\right) \tag{263}$$

So far we have 2 equations and 4 unknowns: I_F, I_R, V_{BE}, V_{BC}. We now use energy conservation from b to ground

$$V_B = R_2 I_B + V_{BE} \tag{264}$$

where by current conservation

$$I_B = I_E - I_C = I_F + I_R - \alpha_F I_F - \alpha_R I_R \tag{265}$$

Thus we get a third equation

$$V_B = R_2\left(I_F + I_R - \alpha_F I_F - \alpha_R I_R\right) + V_{BE} \tag{266}$$

We can now use conservation of energy between collector and ground to get the 4th constraint

$$V_C = R_1 I_C + V_{CB} + V_{BE} = R_1\left(\alpha_F I_F - I_R\right) - V_{BC} + V_{BE} \tag{267}$$

In summary we have a system with 4 equations and 4 unknowns:

$$I_F = I_{ES}\left(e^{\frac{V_{BE}}{v_T}} - 1\right) \tag{268}$$

$$I_R = I_{CS}\left(e^{\frac{V_{BC}}{v_T}} - 1\right) \tag{269}$$

$$V_B = R_2\left(I_F + I_R - \alpha_F I_F - \alpha_R I_R\right) + V_{BE} \tag{270}$$

$$V_C = R_1\left(\alpha_F I_F - I_R\right) - V_{BC} + V_{BE} \tag{271}$$

Using the following standard transistor parameters $I_{ES} = 50 \times 10^{-9}, I_{CS} = 64 \times 10^{-9}, \alpha_F = 0.96, \alpha_R = 0.75, v_T = 0.026$ and the following voltage and resistor values $V_C = V_B = 5, R_1 = 220, R_2 = 10^3$ we get the following

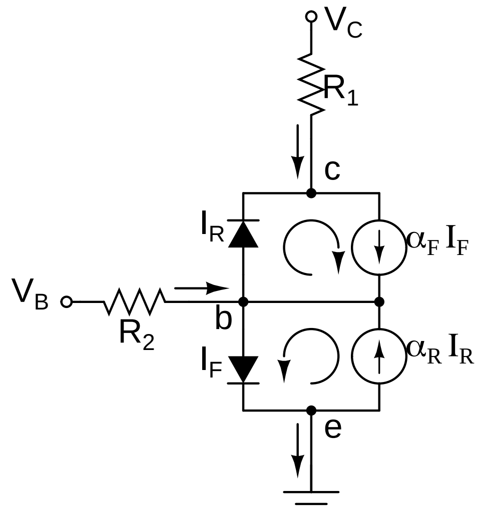

Figure 24: *Example Transistor circuit*

7.12 Example Circuit

Consider the circuit in Figure 28. When the switch is open $I_b = 0$ and thus $I_c = 0$. When the circuit is closed we have $V_{BE} = 0.6$ thus

$$I_{rb} = \frac{10 - 0.6}{1000} = 9.4mA \tag{272}$$

If the transistor were in linear state and assuming a typical $\beta = 100$ then I_c would be 940 mA. The internal resistance of the lamp is $10/0.1 = 100$ Ohms. A current of 940 mA would require a voltage drop of $0.940 \times 100 = 94$ Volts, which is beyond the 10 Volts supplied by the power source. Thus the transistor must be in saturation state. Assuming a typical $V_{CE}^{sat} = 0.1$ V, then the current though the lamp is $(10 - 0.1)/100 = 0.099$. The power dissipated by the resistor is the voltage drop times the current throough the resistor, i.e., $9.4 \times 0.0094 = 88.36$ mWatts. This is within the 1/4 Watts disipation specification of typical resistors.

7.13 Example Circuit: Simple Amplifier, Base Biasing

Plot V_o as a function of V_i. Do we get a logistic?

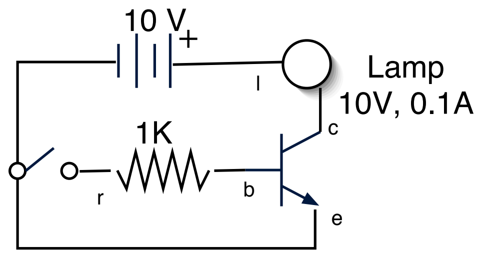

Figure 25: *Example Transistor circuit*

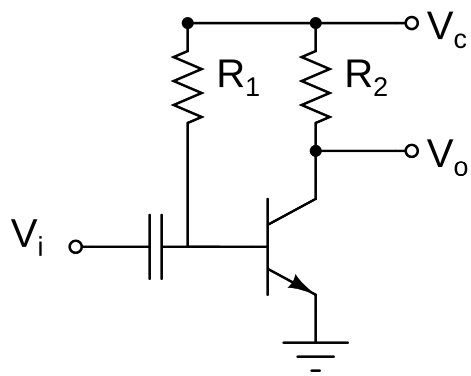

Figure 26: *Base Biasing*

Consider the circuit in Figure 26. Let the average value of V_i be zero (the capacitor in series removes the DC component if not zero). Assuming we operata in the active forward state then as V_b increases I_C increases and thus V_o decreases. As V_i decreases I_c decreases and V_o increases. However note if $R_1 = 0$ then making V_i negative will have no effect on V_o for we enter into the transistors cut-off state. To maximize the range of V_o we want that

$$V_O = V_{CE} = \frac{1}{2}V_C \tag{273}$$

when $V_i = 0$. When in active forward state we have

$$V_C = V_{CE} + R_2 I_C \tag{274}$$

$$2V_{CE} = V_{CE} + R_2 \beta I_B \tag{275}$$

Thus

$$R_2 = \frac{V_{CE}}{\beta I_B} \tag{276}$$

and since

$$I_B = \frac{V_C - 0.7}{R1} \tag{277}$$

Then

$$R_2 = R_1 \frac{V_{CE}}{\beta(V_C - 0.7)} \tag{278}$$

For the base resistor we have

$$V_C = V_{BE} + R_1 I_B \tag{279}$$

Thus

$$R_1 = \frac{2V_{CE} - V_{BE}}{I_B} \tag{280}$$

Standard values for V_{CE} in active mode range in the $[1, 10]$ Volt interval. Thus we want $V_C = 10$ and $V_{CE} = 5$ when $V_i = 0$. Taking the parameters from the 2N2222 $\beta = 100$, $V_{BE} = 0.7$ V. Looking at the Collector Emitter Curve we see a middle range value for I_B is 100 μA

Thus

$$V_C = 10 \tag{281}$$

$$R_2 = \frac{5}{(100)(10^{-4})} = 500\Omega \tag{282}$$

$$R_1 = \frac{10 - 0.7}{10^{-4}} = 93K\Omega \tag{283}$$

While this Analysis is instructive note that the circuit is not very useful for it is highly dependent on a fixed value of β which we know is not a reliable transistor parameter.

7.14 Example Transistor Circuit

Consider the circuit in Figure 27 left. Appendix A, shows the circuit's specification in SPICE code. Figure 27 center shows the voltage at various points in the circuit as a function of time. The right side of the figure shows current as a function of time.

8 The Flip-Flop

8.1 Types of Multi-Vibrators (Flips flops)

- Astable: Continuously changes states (2 unstable states)
- Monostable: One stable state and one unstable state. When a pulse comes the system moves into the unstable state and after a period of time it moves back into the stable state.
- Bistable: Two stable states. When a pulse arrives it changes into another state and remains there until a new pulse arrives.

8.2 State Dynamics

Figure **??** displays a prototypical astable multivabritor (flip-flop) circuit. Figure 29 displays a reinterpretation of the circuit as two mutually connected units. Each unit contains a transistor, a capacitor and 2 resistors.

We will approximate the transistor as a switch that can be into "on", i.e. closed circuit, and "off", i.e., open circuit, states. Jointly there are 4 possible transistor states $(-1, -1), (1, 1), (1, -1), (-1, 1)$.

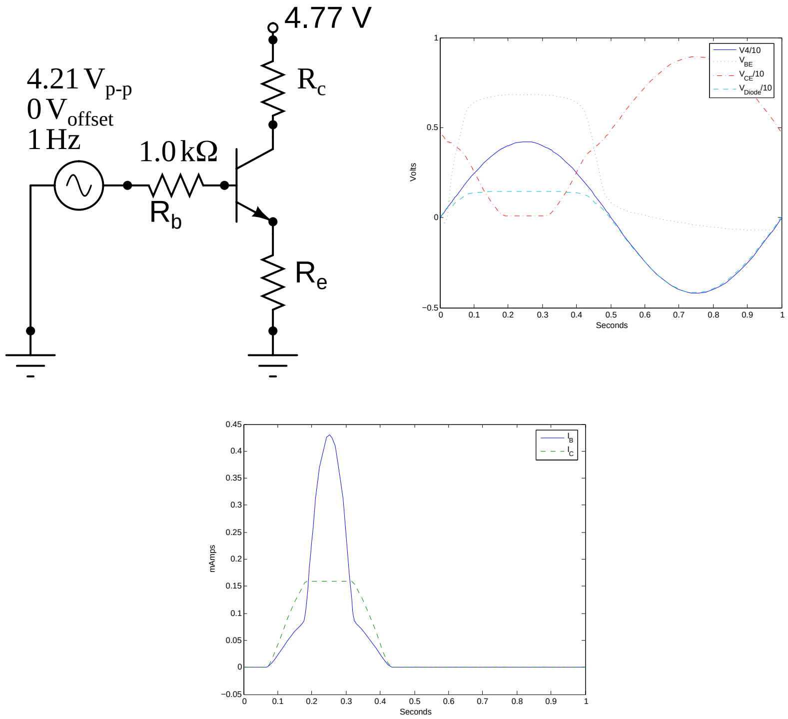

Figure 27: *Example Transistor circuit. The caption $V4$ refers to the alternating voltage source. I_c should say $I_c/100$*

- $S = (1, -1)$: Figure **??** displays an equivalent circuit for the flip-flop with transistor 1 ON state and transistor 2 OFF. Surrounding the circuit are several state variables starting at state $S(-1, 1)$, instantly switching to state $(1, -1)$ and then switching back to $(1, -1)$.

  First observe the current through $Base_1$. It is the sum of the current through R_{21} and the current trhough R_{12}. The reason it does not appear to be so at the beginning of the graph is because at that very moment the second transistor was not in "OFF" state. Note the current through R_{21} decreases exponentially early on due to the charging of C_1 from 1.9 to an assymptote of 2.7 volts.

  The current through $Base_1$ controls the current through Col_1. For simplicity we will use a constant β, model, with the current through Col_1 being approximately 120 times larger than the current thorugh $Base_1$. Finally we investigate how the current trhough Col_1, which acts as a current source, divides up. Let V_{B_1}, I_{B_1} the voltage at and current through the base of transistor . Let I_1 the current through R_{11}, $I - 2$ the current through R_{22}, and V_{C_2} the voltage accross C_2.

$$R_{11}I_1(t) = R_{22}I_2(t) - V_{C_2}(t) \tag{284}$$

$$I_{B_1}(t) = I_1(t) + I_2(t) \tag{285}$$

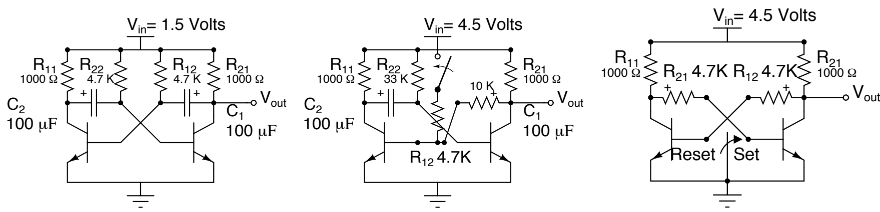

Figure 28: **Left:** *Astable Flip-Flop.* **Center:** Monostable Flip-Flop. **Right:** Bistable Flip-Flop. It has to states (On, or Set, and Off, or Reset). It keeps the state until an external change signal happens.

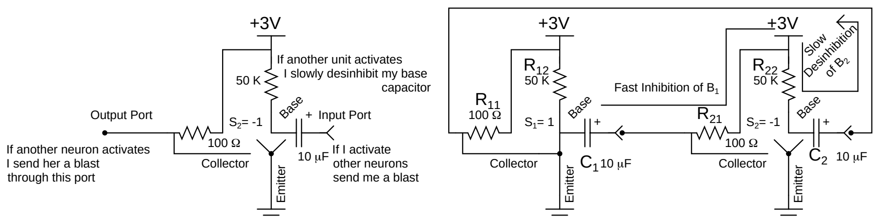

Figure 29: **Left:** Electronic Neuron (Javier). **Right:** Neural Network Interpreation of a Flip-Flop circuit.

Thus xs

$$I_1(t) = \frac{R_{22} I_{B_1}(t) - V_{C_2}(t)}{R_{11} + R_{22}} \tag{286}$$

$$I_2(t) = \frac{R_{11} I_{B_1}(t) + V_{C_2}(t)}{R_{11} + R_{22}} \tag{287}$$

$$V_{B_1} = 3 - R11 \frac{R_{22} I_{B_1}(t) + V_{C_2}(t)}{R_{11} + R_{22}} \tag{288}$$

Note $I_1 \approx I_{B_1}(t)$ and the voltage at B_2 slowly increases as C_2 discharges (note the sign convention for C_2). Now take a look at Figure 30. As the voltage at base 2 approximates 0.5 Volts, the conductance throug collector 2 increases rapidily. As the conductance through collector 2 increases, the current through the base decreases, since the collector 2/ emitter 2 circuit is in parallel with the base 1 /collector 1 line. By equation 288 As the current through 288 decreases, by equation (288), the voltage at the base increases, thus increasing the conductance through Collector 2. This positive feedback loop results on a very rapid increament increment of conductace through the collector, i.e. transistor 2 turns ON and rapid decrease of current through base 1, ie. transistor 1 turns OFF.

- $S = (-1, -1)$: In this case the potential at the base of both neurons is 3 Volts. $V_{B1} = V_{B_2} = 3$ Volts. All else equal the charge of the capacitors will have an effect on which transistor will turn on first. If $V_{C_i} > V_{C_j}$ then unit j will turn on first. If $V_{C_i} = V_{C_j}$ slight differences in the make of the neurons' components will determine which one turns on first.

Figure

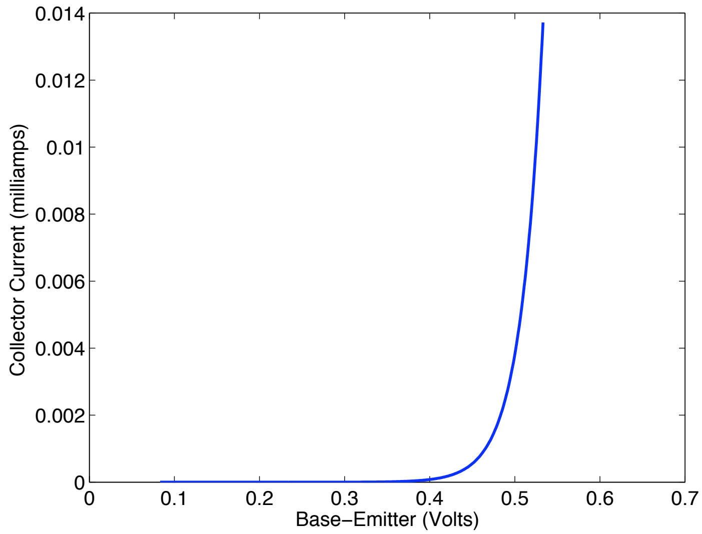

Figure 30: Collector current as a function of base-emitter voltage for a fixed collector-emitter voltage of 10 Volts

9 Domestic Electricity

9.1 Sinusoid current fundamentals

$$\sin(a+b) = \sin a \cos b + \cos a \sin b \tag{289}$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b \tag{290}$$

$$1 = \cos^2 x + \sin^2 b \tag{291}$$

$$\tag{292}$$

From this it follows:

$$\int_0^{2\pi} \cos^2 x dx = \int_0^{2\pi} \sin^2 x dx = \pi \tag{293}$$

$$\int_0^{2pi} \sin x \cos x dx = 0 \tag{294}$$

$$\sin(x+\theta) = \cos\theta \sin x + \sin\theta \cos x \tag{295}$$

We can think of $\sin x$ and $\cos x$ as orthogonal basis for sinusoids of arbitrary phase.

9.2 RMS voltage

Let

$$s(x) = a\sin(x+\theta) \tag{296}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} s(x)dx} = \frac{a}{\sqrt{2}} \tag{297}$$

an AC current with $V_{rms} = k$ produce the same power as a DC with Voltage V.

In the US, $V_{rms} = 120$ Volts $\rightarrow$ Peak voltage is 120* $\sqrt{2} = 169.7$ volts

Power plant produces 3 sinusoids with phase difference of 120 degrees. This 3 lines + grooound line are transported for no more than 300 miles uninterrupted at order 100KVolts. This is transformed down to 7200Volts for transport in residential areas. Each residence grabs 1 line and ground. It converts into 2 lines out of phase by 180 degrees and ground. Voltage difference between each hot line and grounds is 120 volts rms.

Let

$$s(x) = \sin(x + \theta_1) - \sin(x + \theta_2) \tag{298}$$

Then

$$V_{rms}^2(s) = \frac{1}{2\pi}\left(\int_0^{2\pi} s(x)dx\right)^2 = \tag{299}$$

$$\frac{1}{2}[(\cos\theta_2 - \cos\theta_1)^2 + (\sin\theta_2 - \sin\theta_2)^2] \tag{300}$$

$$= 1 + \cos(\theta_2 - \theta_1) \tag{301}$$

In case of $\theta = 120 \rightarrow \cos(120) = 0.5 = 3/2 \rightarrow$ Vrms(s) $= \sqrt{3/2}$ and

$$\frac{Vrms(s)}{Vrms \text{ single phase}} = \sqrt{3} \tag{302}$$

Connecting between two lines out of phase 120 degrees provides sqrt 3 of the single phase voltage $\rightarrow$ 208 Volts in the US

Connecting between two lines out of phase 180 degrees provides 2 times single phase voltage $\rightarrow$ 240 Volts in US

Earth is a relatively good conduntors: average resistance about 100 ohms. For dry earth about 1000 ohms, for rock about 10^7 Ohms.

The resistance to earth of a ground electrode: Let V potential difference between earth and another point. We stick an electrode on earth and measure the current $\rightarrow$ it gives us the resistance to earth of that electrode. In general the deeper, and wider the less resistance. Multiple rods in series also help. This is important for lighting protection. Good resistance is less than 10 ohms. Less than 1 ohm is possible.

10 Appendices

10.1 I: SPICE Code

10.2 Circuit Displayed in Figure 27

```
Spice Code Transistor Circuit

vin1 1 0   DC 4.77
rc 1 2  100
vc 2 3 dc 0
vin2 4 0  AC  SIN(0 4.21 1 1MS 0)
rb 4 5 1K
vb 5 6 dc 0
```

```
q1 3 6 7  Q2N2222A

*2N2222A
*Si 500mW  40V 800mA 300MHz pkg:TO-18 3,2,1
.MODEL Q2N2222A NPN(IS=8.11E-14 BF=205 VAF=113 IKF=0.5 ISE=1.06E-11
+ NE=2 BR=4 VAR=24 IKR=0.225 RB=1.37 RE=0.343 RC=0.137 CJE=2.95E-11
+ TF=3.97E-10 CJC=1.52E-11 TR=8.5E-8 XTB=1.5 )

ve 7 8 dc 0

diod1  8 9 diodemod
.model diodemod d n=2
vd 9 10 dc 0
re 10 0 100
```

10.3 II: Xcircuit

The diagrams in this document have been made with XCircuit. Here are some tips

- click on library icon on right side bar. Click on desired symbol. To select item click and hold until yellow To draw lines simply click. To end drawing line "option click". To erase last step "command click".
- To write file: File -¿ Write X circuit PS: It produces a postscript file that can be open and edited using xcircuit.
- To select group: "option click"
- To draw a connecting dot press the dot in the keyboard
- To write stuff: text -¿ make labels. You can then use text-¿ style to do subscript or text -¿ font for greek symbols

10.4 III: Operating Parameters of PN222A Switching transistor

- Collector Emitter Breakdown Voltage: 40 V
- Collector Base Breakldown Voltage: 75 V
- Emitter-Base Breakdown Voltage: 6 V
- Collector Breakdown current: 1 A
- Collector Cutoff Current: 10 nA
- Emitter Cutoff current 10 μ A
- Base Cutoff current 20 μ A
- $h_F E$: 35 to 300
- V_{CE}^{sat}: 0.3 to 1.0 V
- V_{BE}^{sat}: 0.6 to 2.0 V
- Power Dissipation: 0.625 Watts. $I_c V_{CE}$ must be less than P_{Dmax}

10.5 III: Operating Parameters of 2N3904 Switching transistor

- Collector Emitter Breakdown Voltage: 40 V
- Emitter-Base Breakdown Voltage: 6 V
- Collector Breakdown current: 200 mA

E	B	C		E	B	C		E	B	C		E	B	C
-	+			+	-				+	-			-	+
0.7 V				Open Loop				0.7 V				Open Loop		

Table 6: Testing a Transistor using Diode Function of Digital Meter. First row: Emitter, Base, Collector. Second Row: Location of the Positive and Negative leads of meter. Third row: Expected Result.

- Max Power Dissipation: 500 mW
- Collector Cutoff Current: < 50 nA
- Emitter Cutoff current $< 50\ \mu$ A
- $h_F E$: 30 to 300.

I_c	min h_{FE}	max h_{FE}
0.1m A	60	-
1m A	80	-
1m A	80	300
50m A	60	-
100m A	30	-

- V_{CE}^{sat}: ≤ 1.6 V. Spice uses 0.07 V for model of this transistor.
- V_{BE}^{sat}: ≤ 0.95 V. Spice uses 0.7 V for model of this transitor.
- Max Power Dissipation: 500 mW

10.6 IV: Testing Transistor Multimiter Diode Test

When set to diode test function the meter provides enought voltage to forward bias and reverse bias a transistor. A good transistor shall respond as in Table **??**

10.7 V: Vocabulary

- Voltage Source: Delivers constant voltage regardless of load. Quite remarkable but very common in nature.
- Current Source: Delivers constant current regardless of load. Much harder to find in nature. The simplest current source approximation is a large voltage source followed by a large resistance. Collector-Emmitter in transistor can be seen as a current source. For a given base current it delivers constant current between collector and emitter regardless of the voltage drop between collector and emitter.

References

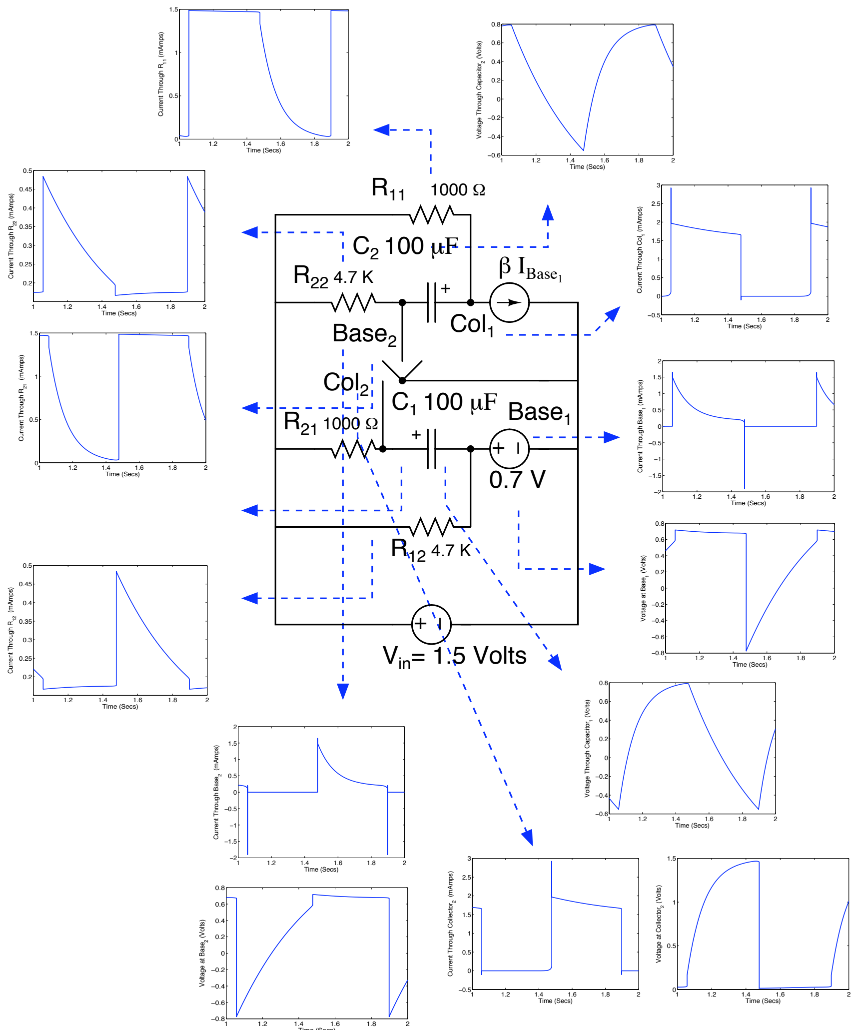

Figure 31: **Left:** *Equivalent Circuit for Flip-Flop with first transistor in "ON" state, and second transistor in "OFF" state.*

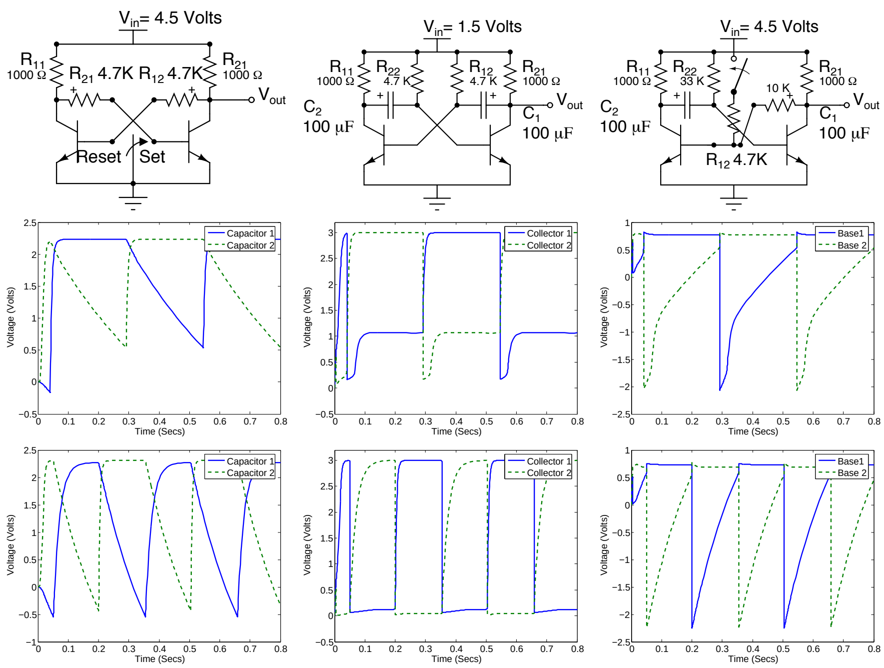

Figure 32: **First Row Parameters:** *R1Inhibit2 100 Ohms, R1Desinhibit1 50K, C1 10u IC=0.001V, R2Inhibit1 100 Ohms, R2Deshinhibit2 50K, C2 10u IC=0.0V.* **Second Row Parameters:** *R1Inhibit2 50 Ohms, R1Desinhibit1 5K, C1 100u IC=0.001V, Q1 Q2N3904, R2Inhibit1 50 Ohms, R2Deshinhibit2 5K, C2 100u IC=0.0V,* **Third Row Parameters:** *R1Inhibit2 200 Ohms, R1Desinhibit1 10K, C1 20u IC=0.001V, Q1 0 Q2N3904, R2Inhibit1 1K, R2Deshinhibit2 10K, C2 20u IC=0.0V,*