CSE 12:

Basic data structures and object-oriented design

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Lecture Ten
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Linear data structures: asymptotic time costs

- Let's review the "score card" of the ADTs we've covered so far.
- Let's consider three fundamental operations:
  - `void add (T o);`
  - `void remove (T o);`
  - `T find (T o);`
    Search for an element in the container that `equals o` and returns it; if no such object exists, then returns `null`.

Array-list and linked-list scorecard

	Array-list	Linked-list	
`add(o)`	$O(1)$	$O(1)$	Adding is fast.
`find(o)`	$O(n)$	$O(n)$	Finding is slow.
`remove(o)`	$O(n)$	$O(n)$	Removing is slow.

Array-list and linked-list scorecard

- There are many occasions where the user will *add* new data relatively *rarely*, but want to *find* data already in the data structure relatively *frequently*.
- In order to improve the asymptotic time cost of the **`find(o)`** and **`remove(o)`** operations, we will make use of order relationships between data elements.
  - Once we've *found* an element within a data structure, it is typically easy for the data structure to *remove* it.

Why `find` something?

- It may strike some as odd that an ADT would support the method `T find (T o)`.
- After all, if the user knows the object `o` he/she is looking for, then why call `find` at all?
- *Answer*: sometimes the user knows *part* of the information about an object `o`, but does not have the whole record.
  - This illustrates the difference between a record's *key* and its *value*.

Keys and values

- The part of the **`Student`** object that the user always knows is called the *key* (e.g., student ID number at Student Health).
- The rest of the **`Student`** record is called the *value*.

```
class Student {
  String _studentID;                          Key
  String _firstName, _lastName;
  String _address;                            Value

  Student (String studentID) {
    _studentID = studentID;
  }

  Student (String studentID, String firstName, String lastName,
           String address) {
    _studentID = studentID;
    _firstName = firstName;
    _lastName = lastName;
    _address = address;
  }
}
```

Keys and values

- The user may store many **`Student`** objects inside a **`List12`** container, e.g.:

```
list.add(new Student("A123", "Bill", "Carter", "123 Main St"));
list.add(new Student("A213", "Jimmy", "Clinton", "124 Main St"));
...
list.add(new Student("B092", "Hillary", "Nixon", "125 Main St"));
```

- Later, the user may wish to find a particular **`Student`** object using just the key, e.g., the student ID:

```
final Student cse12Student = list.find(new Student("A123"));
```

Student containing both the key and value.

Student initialized with just the key.

Finding a particular key

- Given a request to find a particular key, and given that keys often have an *order relation* defined between them, it seems silly to search through the container *as if the keys were all unrelated*.

- *Example*: Suppose we are searching for the student ID "`C237`". Do we really need to start at the very beginning?

Search

`A101`	`B972`	`D192`
`A102`	`C092`	`...`
`A125`	`C100`	
`A192`	`C200`	
`A204`	`C203`	
`B135`	`C237`	
`B193`	`C292`	

No -- the *natural order* among *keys* imposes structure on the "search problem" that lets us find a particular key much more quickly.

Binary order relations

- An example of a binary order relationship is the Java < operator, e.g.:

```
int  a = 3, b = 4;
if (a < b) {
   ...
}
```

- However, the < operator is only valid on primitive numeric variables (**`int`**, **`float`**, **`double`**, etc.).

Binary order relations

- More generally, two Java **`Objects`** can be compared if they are **`Comparable`**, using the **`compareTo`** method: **`int compareTo (T o);`**
- **`o1.compareTo(o2)`** is:
    - < 0 if **`o1`** is “less than” **`o2`**
    - == 0 if **`o1`** is “equal to” **`o2`**
    - > 0 if **`o1`** is “greater than” **`o2`**
- Classes that implement the **`compareTo(o)`** method can implement the **`Comparable<T>`** interface.

Comparable<T>

- Example:

```
class Student implements Comparable<Student> {
  ...
  int compareTo (T other) {
    return _studentID.compareTo(
      other._studentID
    );
  }
}
```

In this particular case, we can just delegate to the `String.compareTo(o)` method, since `String` implements `Comparable<String>`.

Faster search using recursion

Searching a sorted list

- How will defining this "ordering relation" using `Comparable<T>` help us to find a key more quickly?
- Let's consider a simpler example in which we wish to find an integer within a *sorted* list of numbers.
- We will implement a method

```
int search (int[] numbers, int targetNum,
            int startIdx, int endIdx);
```

which will search through an array of `numbers`, starting at the `startIdx` and ending at the `endIdx`, looking for the `targetNum`.

Searching a sorted list

- Consider the following example:

  `search(numbers, targetNum, startIdx, endIdx)`:

  where

```
int targetNum = 79;

int startIdx = 0;
int endIdx = 15;

int[] numbers = {
  16, 26, 31, 40, 43, 45, 51, 55, 58, 67, 69, 73, 79, 87, 88
};
```

- What is the optimal search strategy given that numbers is already sorted?

Binary search

- The optimal search strategy (minimum time cost) for a list of sorted elements is *binary search*.
  - The search is *binary* because we repeatedly divide the list into 2 *pieces*.
- Search algorithm:

```
Pick a guessIdx = (startIdx + endIdx) / 2;
if (numbers[guessIdx] == targetNum) {
  return guessIdx;
} else if (numbers[guessIdx] < targetNum) {
  Search the "right half" of the list for targetNum.
} else {
  Search the "left half" of the list for targetNum.
}
```

16, 26, 31, 40, 43, 45, 51, 55, 58, 67, 69, 73, 79, 87, 88

Binary search

- Let's look for **`targetNum=79`**.
- Search algorithm:

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}
```

Done in 4 guesses!

`16, 26, 31, 40, 43, 45, 51, 55, 58, 67, 69, 73, 79, 87, 88`

Binary search and recursion

- Binary search is a classic example of a *recursive algorithm*:
  - The algorithm makes repeated *calls to itself* to get its work done, e.g.:
    "Search algorithm:

    ...

    ```
    Search the "right half" of the list for targetNum.
    ```
    "
  - Each *recursive call* operates on a smaller problem than the original (e.g., it searches only half the list).
  - Eventually, the algorithm operates on a trivial input size (e.g., a list of 1 element) and terminates.

Recursive binary search

- Let's return to our example of searching through an array **`numbers`** of sorted integers for a particular **`targetNum`**.
- Search algorithm:

```
// Assume targetNum is always somewhere inside numbers
int search (int[] numbers, int targetNum, int startIdx, int endIdx) {
  int guessIdx = (startIdx + endIdx) / 2;
  if (numbers[guessIdx] == targetNum) {
    return guessIdx;
  } else if (numbers[guessIdx] < targetNum) {
    return search(numbers, targetNum, guessIdx+1, endIdx);
  } else {
    return search(numbers, targetNum, startIdx, guessIdx-1);
  }
}
```

Base case

Recursive part

Binary search and recursion

- The worst-case time cost of binary search depends on *how many times* the list can be *divided in half*.
- `int length = endIdx - startIdx + 1;  // 16`

16
divide in half
8
divide in half
4
divide in half
2
divide in half
1

Binary search and recursion

- The worst-case time cost of binary search depends on *how many times* the list can be *divided in half*.
- `int length = endIdx - startIdx + 1;  // 16`

```
16
    divide in half
8
    divide in half
4
    divide in half
2
    divide in half
1
```

$\log_2 16 = 4$ times

- If the list has n elements, then binary search has a worst-case time cost of $O(\log n)$.
  - Huge improvement over $O(n)$.

Binary search and objects

- What if we want to execute binary search on a list of objects?
  - This is easy if the objects are **`Comparable`**.
- Search algorithm (to find object **`o`**):

```
Pick a guessIdx = (startIdx + endIdx) / 2;
if (objects[guessIdx].compareTo(o) == 0) {
  return guessIdx;
} else if (objects[guessIdx].compareTo(o) < 0) {
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Albert
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```

Done

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Sorting and recursion

- Recall, however, that binary search requires the list to have been *already* sorted.
  - How was this accomplished?
- It turns out that the fastest sorting algorithms are implemented using *recursion*:
  - For instance, the MergeSort algorithm (next week) successively divides a list of ordered elements into two halves, sorts them separately, and then combines the results.

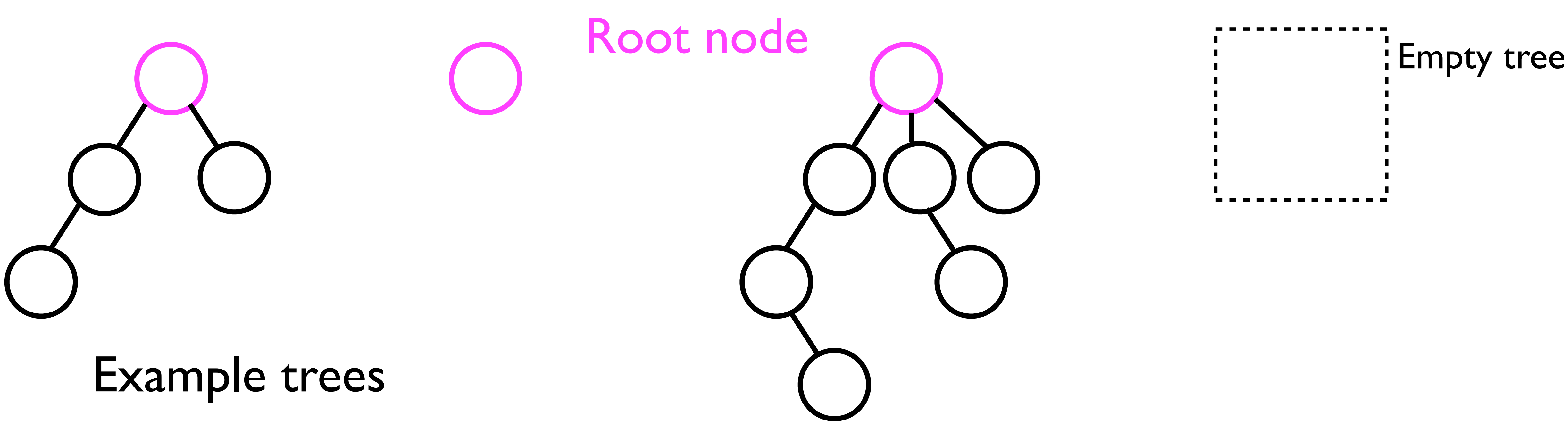
Data structures and recursion

- Even though a sorted list of data is useful, what happens if we want to add more data into the list? How do we *keep* the data in sorted order?
  - Using a list in these cases will be inefficient.
  - More efficient is a *tree-based* data structure.
    - Trees are *non-linear* data structures because each element may be adjacent to more than 2 other elements.
    - Trees are *recursive data structures* -- each "branch" of a tree forms a "tree" in itself.

Binary Trees

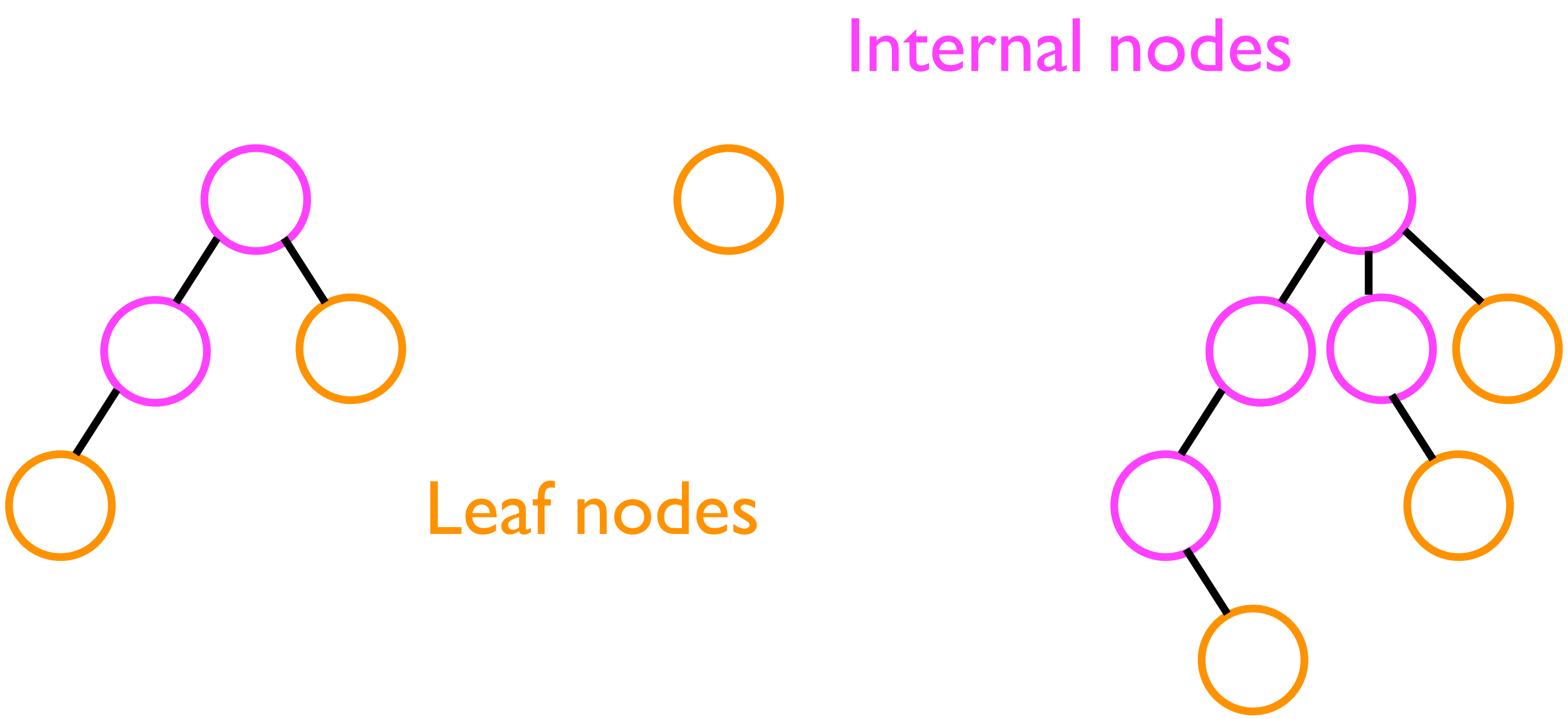
Trees

- A *tree* is an interconnected set of *nodes* that are organized in a hierarchy.
  - There is one node labeled the *root* of the tree.
  - Every node except the root has exactly 1 *parent* node.
  - Each node may have 0 or more *child* nodes ("children").
    - Cycles are prohibited -- only one path may exist between any pair of nodes.
  - Parents and children are connected by *edges*.



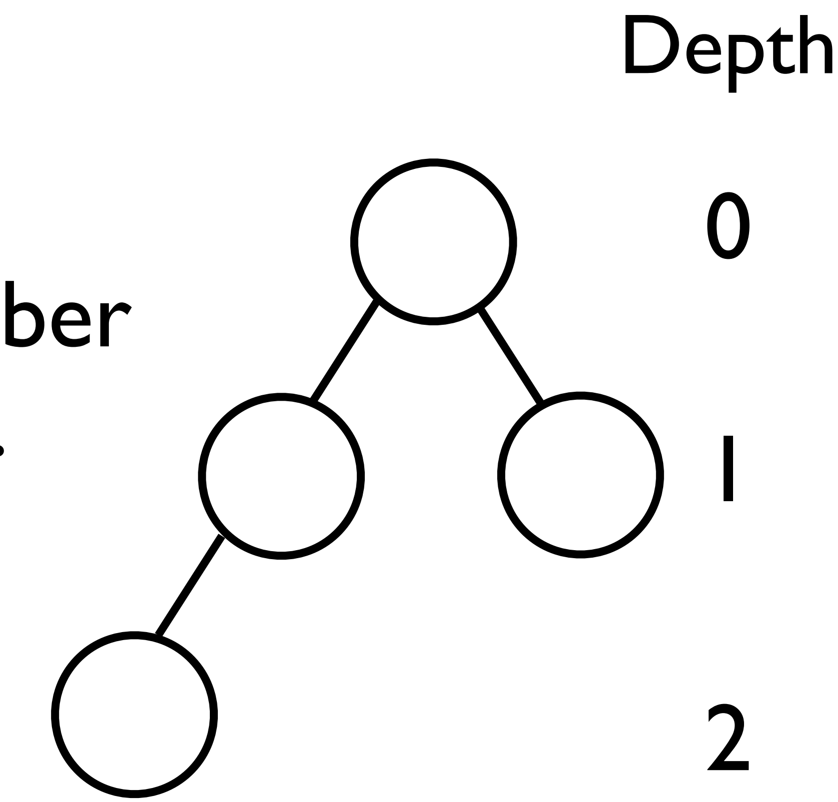
Trees

- A node with no children is called a *leaf*.
- A node with at least one child is called an *internal node*.



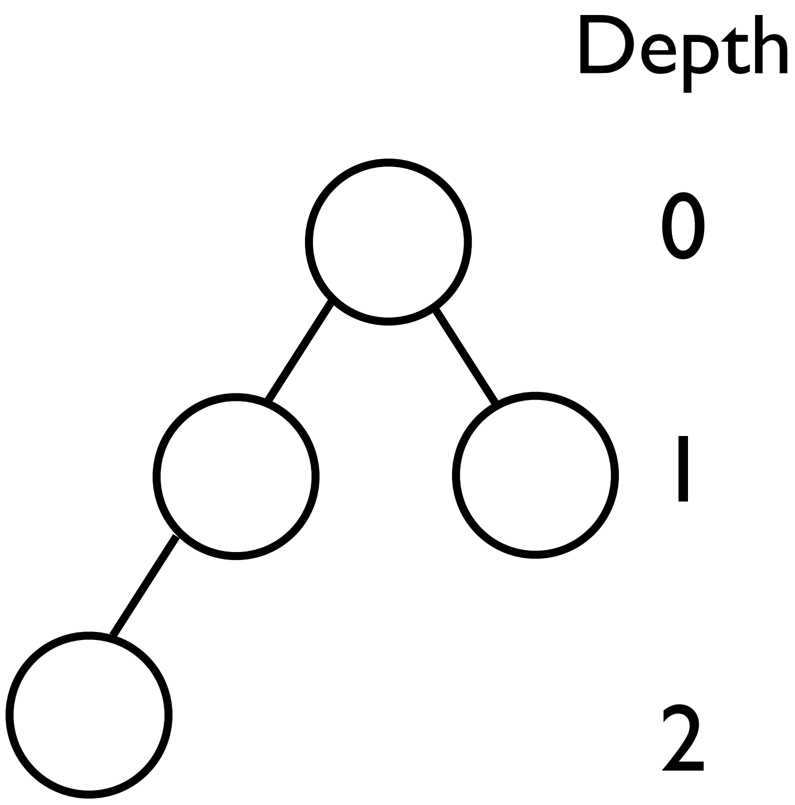
Depth, height, and level

- Depth (iterative definition):
  - The *depth* of a node *n* is the number of edges between *n* and the root.
  - The root has depth 0.



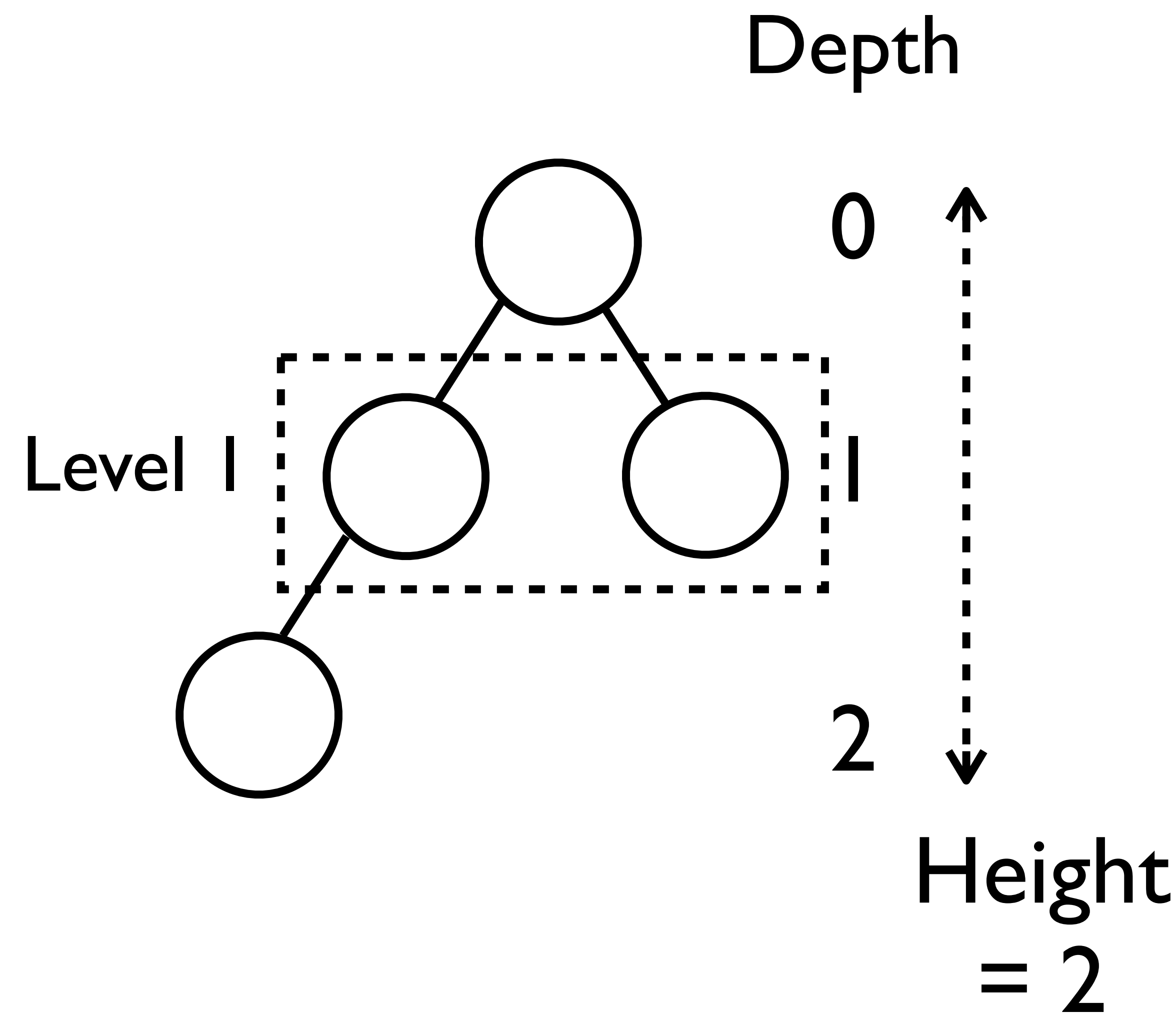
Depth, height, and level

- Depth (recursive definition):
  - Base case: The depth of a node *n* is 0 for the root; or
  - Recursive part: 1 + the depth of *n*'s parent node.



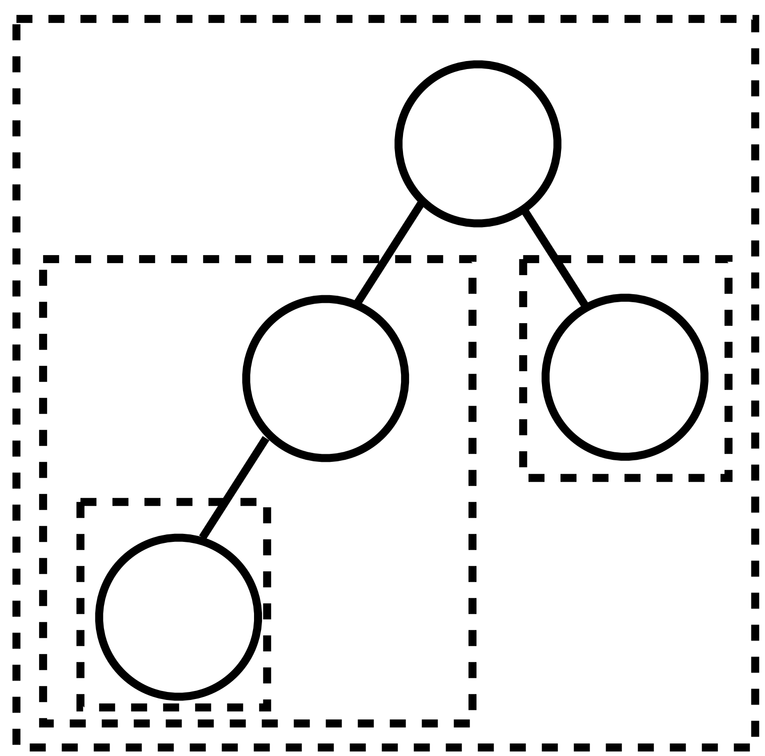
Depth, height, and level

- The *height* of a tree *T* is the maximum depth of any node in the tree.
  - Equivalent to length of longest path from the root to any leaf.
- A *level* of the tree consists of all the nodes at a particular depth.



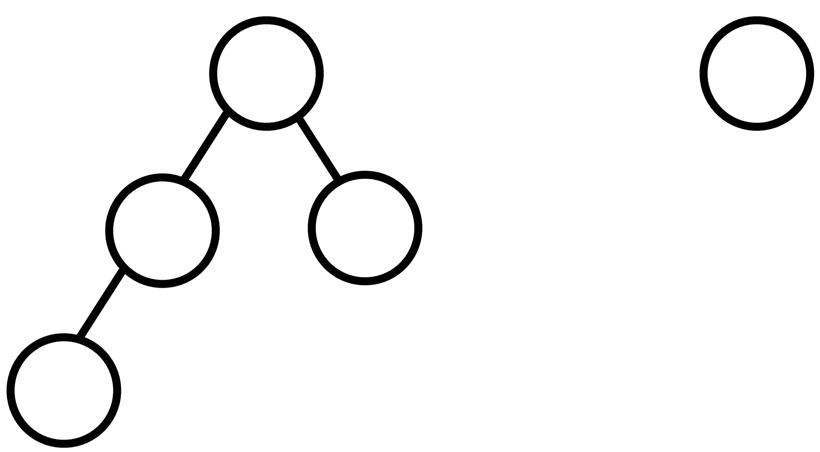
Sub-trees

- Each node in a tree is the *root* of its own *sub-tree*.
- The gray boxes below show all possible sub-trees.

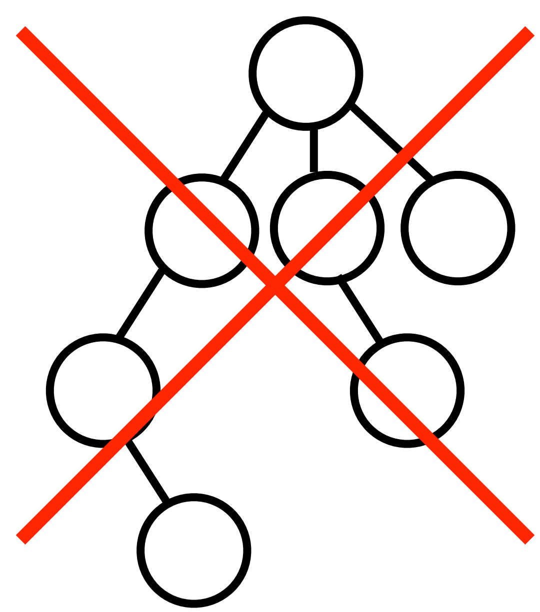


Binary trees

- A *binary tree* is a tree in which every node has at most 2 children.



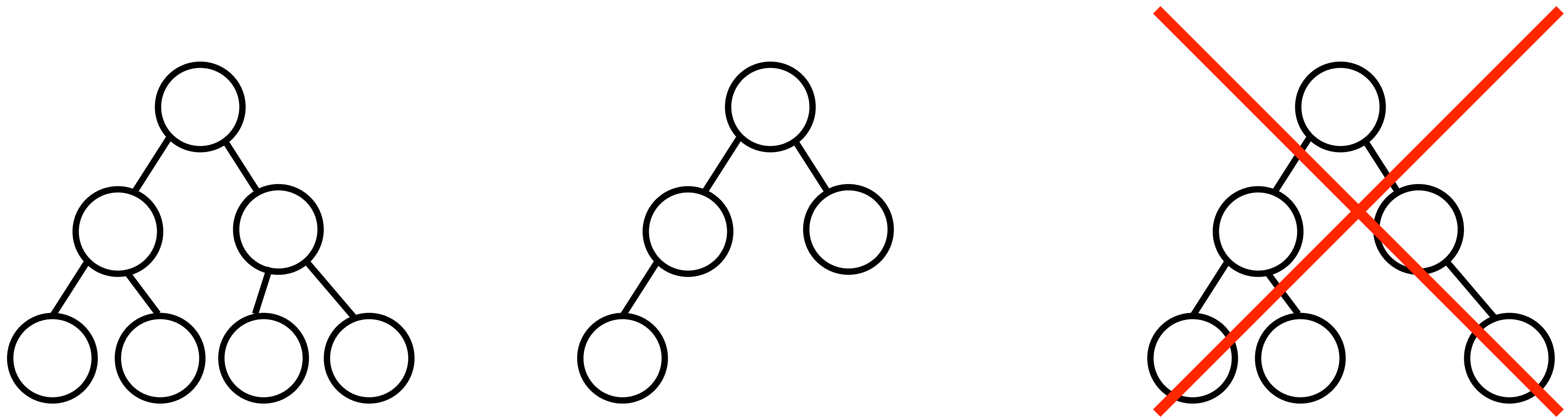
Examples of binary trees



Not a binary tree

Binary trees

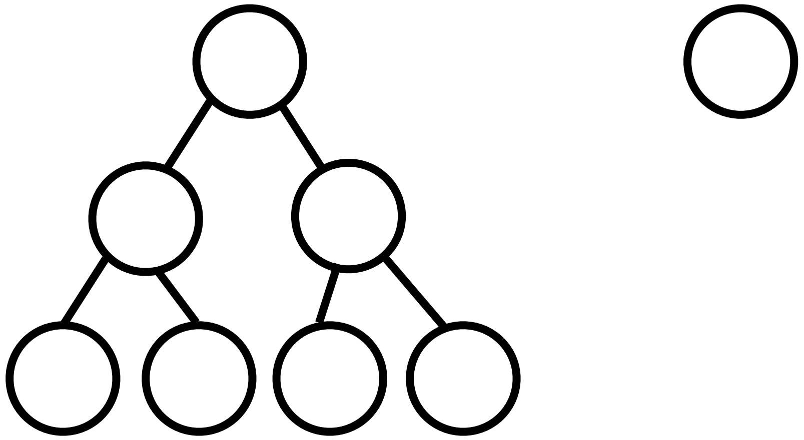
- A binary tree is *complete* if every level of the tree is completely filled except possibly the last *and* the last level is (partially) filled from left to right.



Complete Complete Not complete

Binary tree properties

- A binary tree of height *h* is *full* if every node at depth $d < h$ has 2 children.



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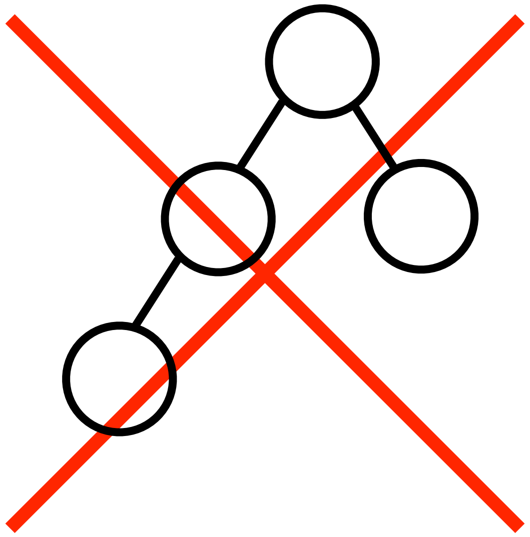


Not a full binary tree

Binary tree properties

- A full binary tree with height *h* has 2^h leaf nodes and 2^{h+1} -1 nodes in total.
- Conversely, a full binary tree with *n* nodes total has height $\log_2(n+1)-1$.

Binary tree properties

- More generally, a binary tree *T* (not necessarily full) with *n* nodes has:
  - Minimum height $\log_2(n+1) - 1$ (when *T* is full).
  - Maximum height n-1 (when *T* is just a "chain" of nodes in which no node has more than 1 child).
- Why important?
  - The time cost of important tree operations such as **`find(o)`** depend on the average/maximum height of an arbitrary node in the tree.

Tree nodes

- Like nodes in a linked list, nodes in a tree contain a *data element* (otherwise, trees would be useless for ADTs).
- However, nodes in a tree contain more than 2 “links” (edges) to other nodes.
  - One link to parent node.
  - One link to each child node.

Node class for general trees

- From this description, we can create a **`Node`** class for use in *general* trees (**not** for P3!):

```
class Node<T> {
  Node<T> _parent;  // link to parent node
  Node<T>[] _children;  // links to children
  int _numChildren;
  T _data;  // data element the node stores
}
```

- Alternatively, we can used a *linked list* to manage the child **`Node`**s:

```
class Node<T> {
  Node<T> _parent;  // link to parent node
  LinkedList<T> _children;  // links to children
  T _data;  // data element the node stores
}
```

Node class for binary trees

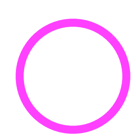
- From *binary* trees, we can define a `Node` more simply:

```
class Node<T> {
  Node<T> _parent;
  Node<T> _leftChild, _rightChild;
  T _data;
}
```

Defined to be `null` if child does not exist.

- We can then begin creating `Nodes` and assembling a tree:

```
final Node<String> root = new Node<String>();
root._leftChild = new Node<String>();
root._rightChild = new Node<String>();
root._rightChild._leftChild = new Node<String>();
```



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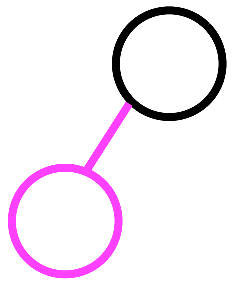

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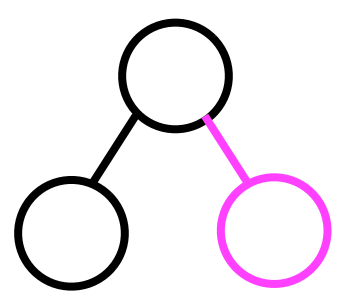

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```
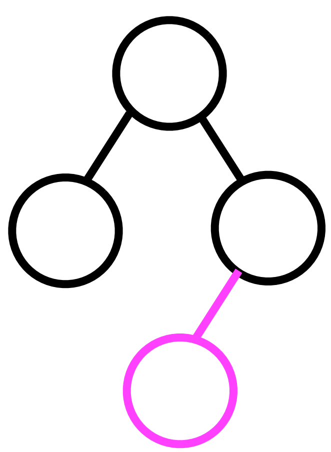



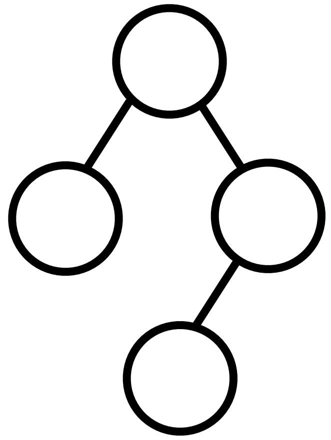
Node class for binary trees

- From *binary* trees, we can define a **`Node`** more simply:

```
class Node<T> {
  Node<T> _parent;
  Node<T> _leftChild, _rightChild;
  T _data;
}
```

- We can then begin creating **`Nodes`** and assembling a tree:

```
final Node<String> root = new Node<String>();
root._leftChild = new Node<String>();
root._rightChild = new Node<String>();
root._rightChild._leftChild = new Node<String>();
```



Tree operations

- We will consider two fundamental operations:
  - **`add (o, parent, leftOrRight)`** -- add a new node (containing the object o) as the leftOrRight child of the specified parent.
  - **`find (o)`** -- find and return the **`Node`** containing data **`o`**.
- Note that these operations will be used *internally* by ADTs we develop *based on* trees.
  - This is why we find and return the *node* instead of the data contained *inside* the node.
  - They will *not* be exposed to the user of, say, the Heap ADT, which is built using a binary tree.

Adding a node

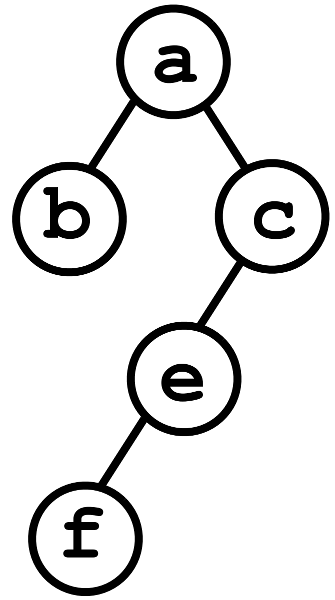
- Given the parent node, it is straightforward to add a new node that is either the left or right child of the parent:

```
void add (T o, Node<T> parent,
          boolean isLeftChild) {
  final Node<T> node = new Node<T>();
  node._data = o;
  if (isLeftChild) {
    parent._leftChild = node;
  } else {
    parent._rightChild = node;
  }
}
```

Finding a node

- Finding a node in a binary tree is best implemented using recursion. We'll let `root` represent the root of the *sub-tree* we are currently searching.

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```



Finding a node

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  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```

Combined assignment to `node` and comparison to null. This is compact notation, but it sometimes can also yield more readable code.

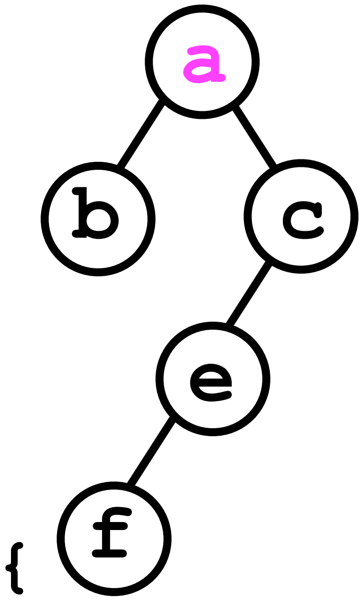
Finding a node

- Watch how the method works for `find(a, "e")`:

root: a

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```

No

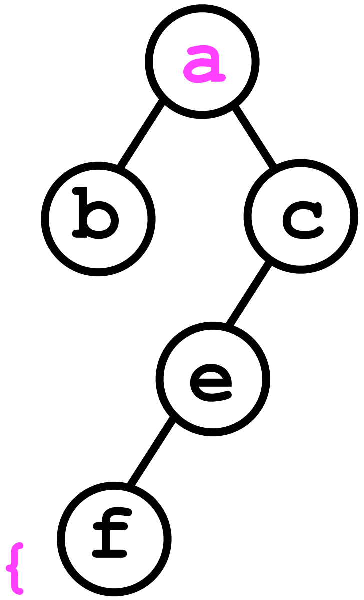


Finding a node

- Watch how the method works for `find(a, “e”)`:

root: a

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```



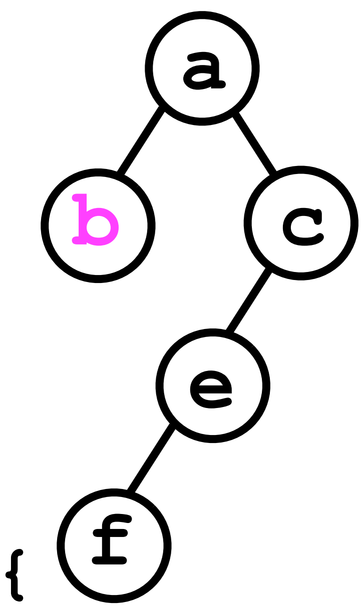
Finding a node

- Watch how the method works for `find(a, “e”)`:

root: b

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```

No

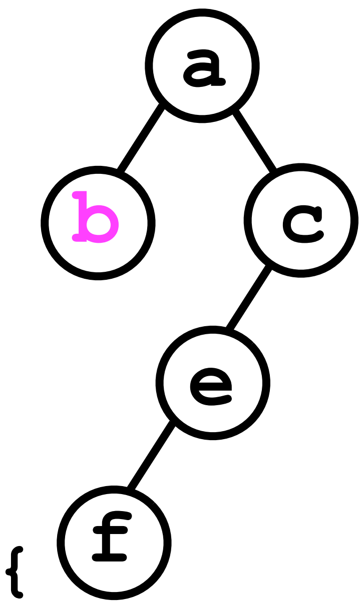


Finding a node

- Watch how the method works for **`find(a, "e")`**:

root: b

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```

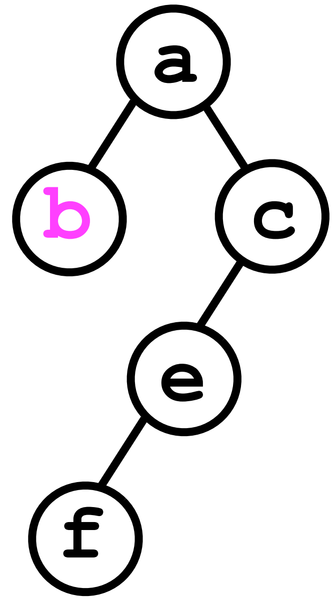


Finding a node

- Watch how the method works for **`find(a, "e")`**:

root: b

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```

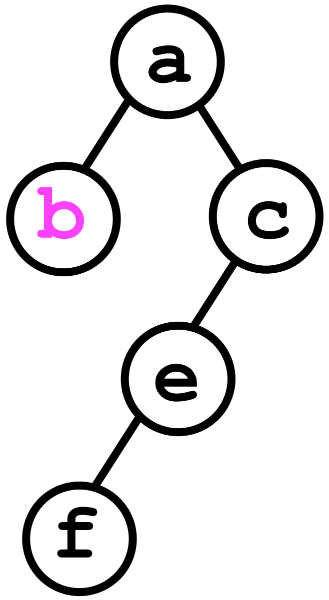


Finding a node

- Watch how the method works for `find(a, "e")`:

root: b

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```

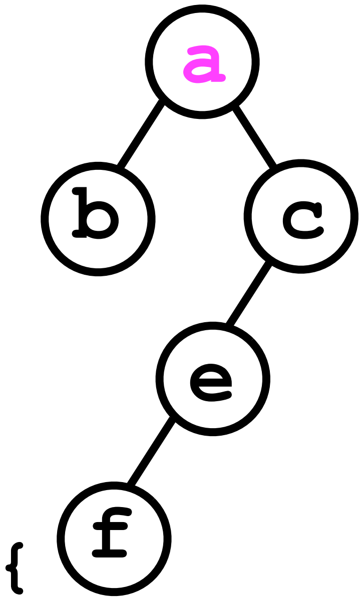


Finding a node

- Watch how the method works for `find(a, “e”)`:

root: a

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```

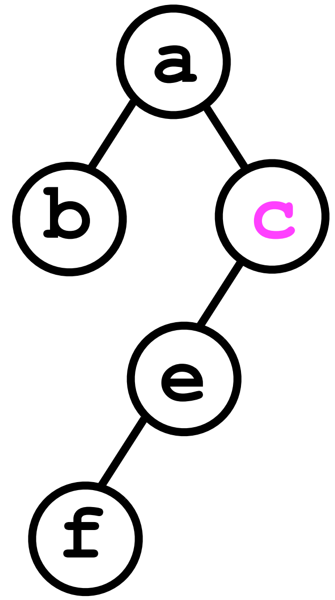


Finding a node

- Watch how the method works for `find(a, "e")`:

root: c

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {        No
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```

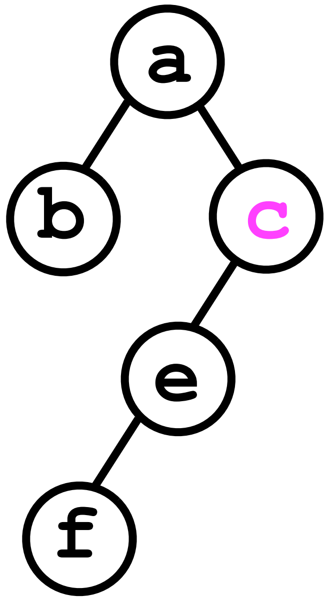


Finding a node

- Watch how the method works for `find(a, "e")`:

root: c

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```



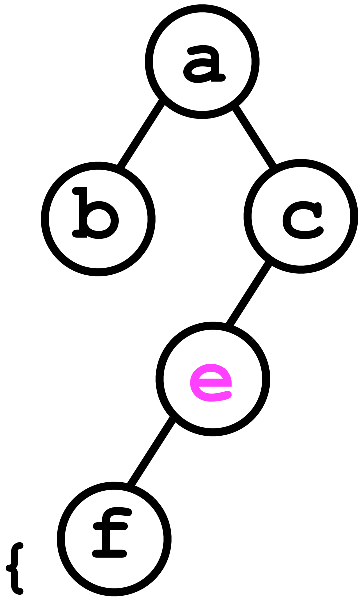
Finding a node

- Watch how the method works for `find(a, "e")`:

root: e

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```

YES!!!!!!!!!!!!!!!!!!!!!!!!!!!!



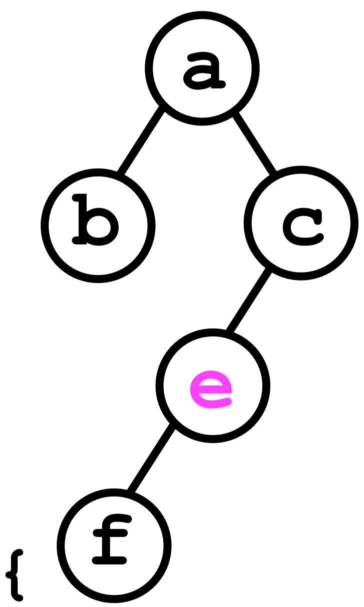
Finding a node

- Watch how the method works for `find(a, “e”)`:

root: e

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```

The returned node will “propagate back up” the recursive calls.

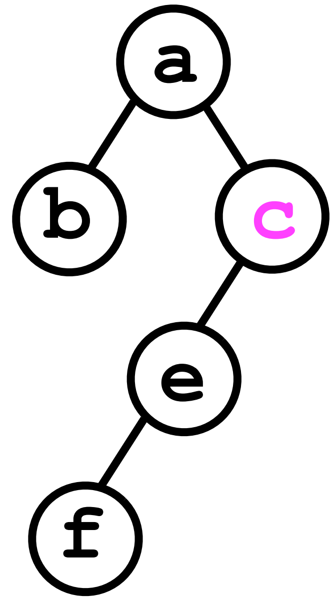


Finding a node

- Watch how the method works for `find(a, "e")`:

root: c

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```

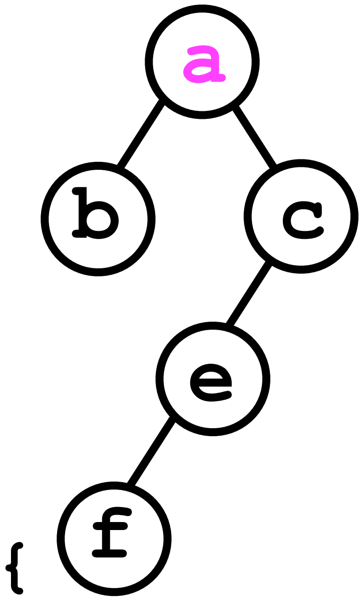


Finding a node

- Watch how the method works for **`find(a, "e")`**:

root: a

```
Node<T> find (Node<T> root, T o) {
  if (root._data.equals(o)) {
    return root;
  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```



Finding a node

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  if (root._data.equals(o)) {
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  }
  Node<T> node;
  if (root._leftChild != null &&
      (node = find(root._leftChild, o)) != null) {
    return node;
  } else if (root._rightChild != null &&
      (node = find(root._rightChild, o)) != null) {
    return node;
  } else {
    return null;
  }
}
```
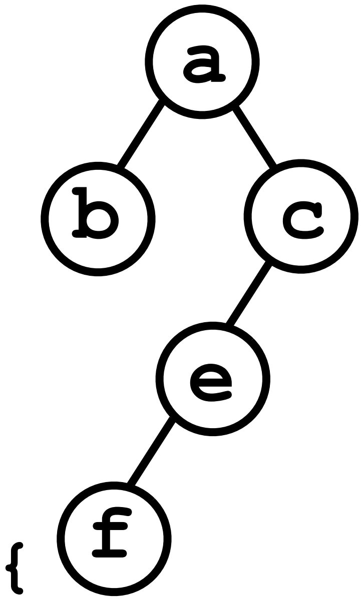



Done!!!!!!!!!!!!!!!!!!!!
!!!!!!!!!!!!!!!!!!!!!

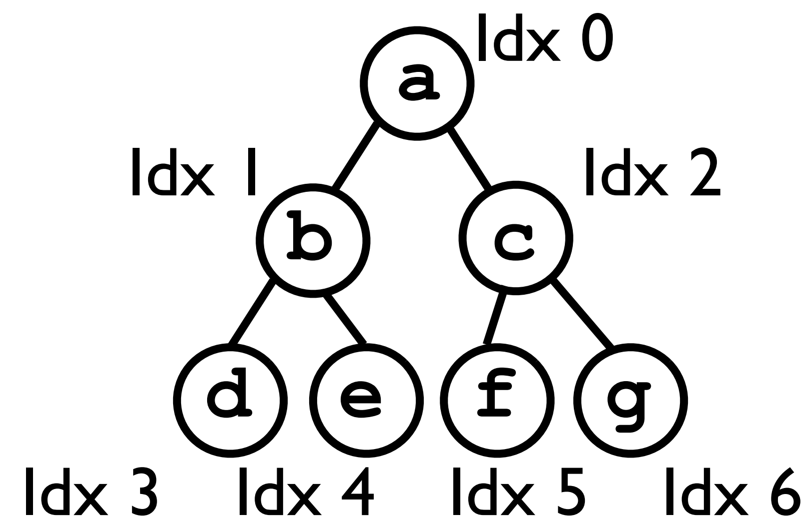
Array-based binary trees.

Array-based binary trees

- Just as *lists* can be implemented by either a linked chain of `Node`s or an array, a *binary tree* can be implemented as a tree of `Node`s or an array as well.

- Each "node" in the tree will be assigned a unique index at which its *data* should be stored.

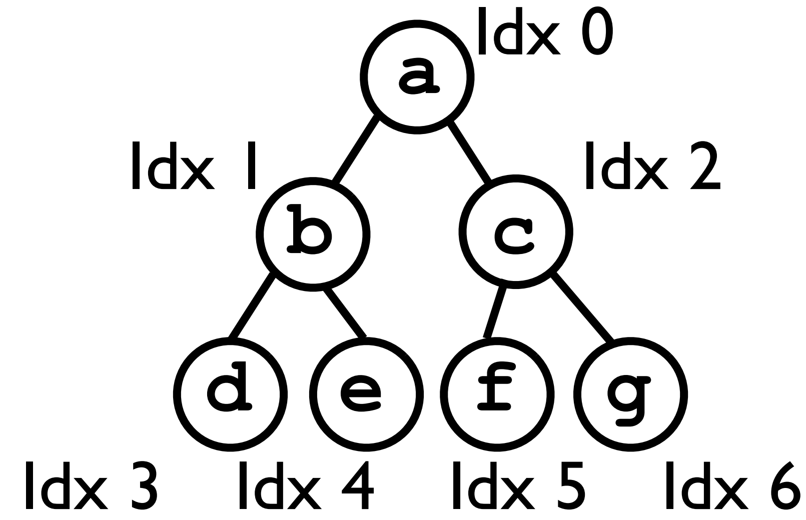
- Given the index of a particular "node", the index of its *parent*, and the indices of its *children*, can be easily computed.



a	b	c	d	e	f	g
0	1	2	3	4	5	6

Array-based binary trees

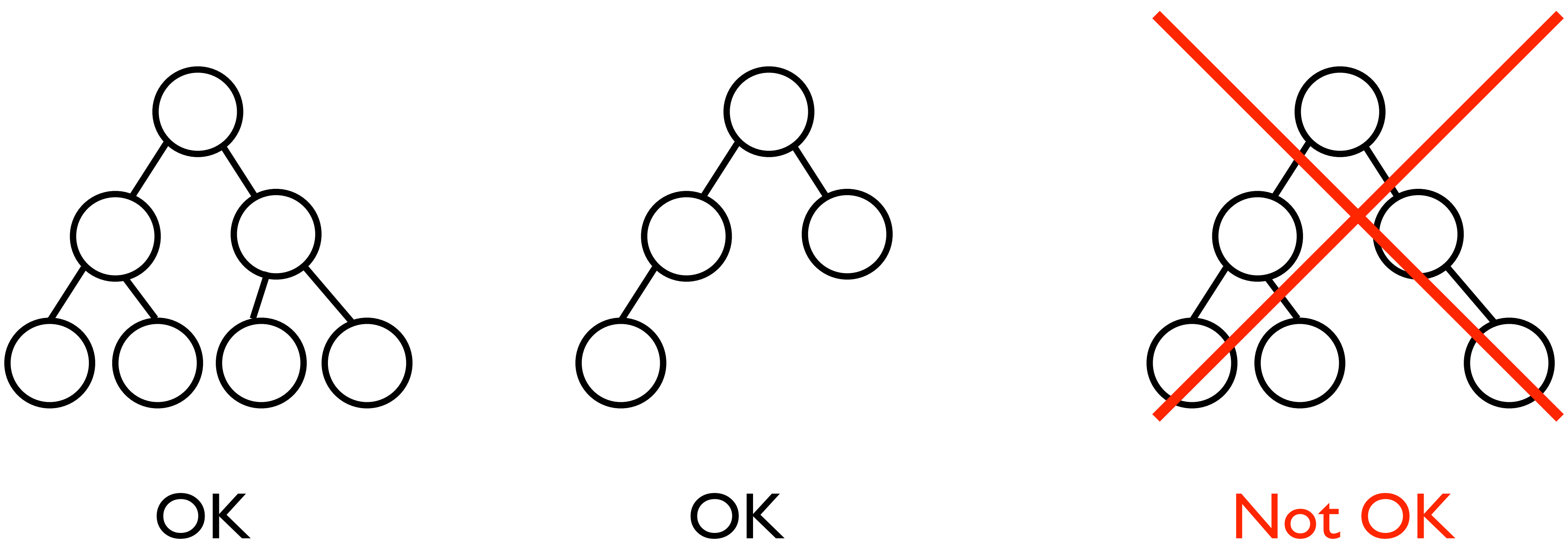
- The index(*n*) of a node *n* with parent *p* is:
  - 0 if *n* is the root node.
  - 2*index(*p*)+1 if *n* is left child of *p*.
  - 2*index(p)+2 if n is right child.
- The parentIndex(**`idx`**) of a node stored at **`idx`** is **`(idx-1)`**/2.
- Examples:
  index(c) = 2*index(a)+2 = 2*0+2 =1
  parentIndex(4) = (4-1)/2 = 1.5 = 1.



a	b	c	d	e	f	g
0	1	2	3	4	5	6

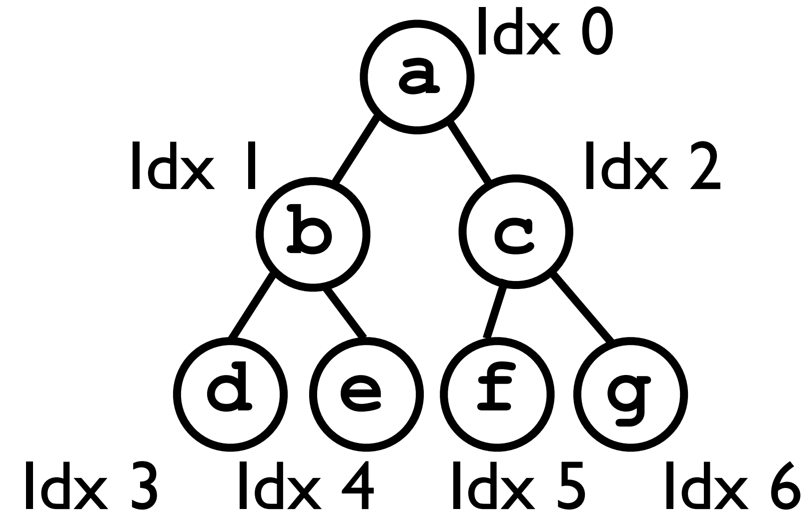
Array-based binary trees

- Note that this array-based representation applies only to *complete* binary trees.
    - A binary tree is *complete* if every level of the tree is completely filled except possibly the last *and* the last level is (partially) filled from left to right.



Array-based binary trees

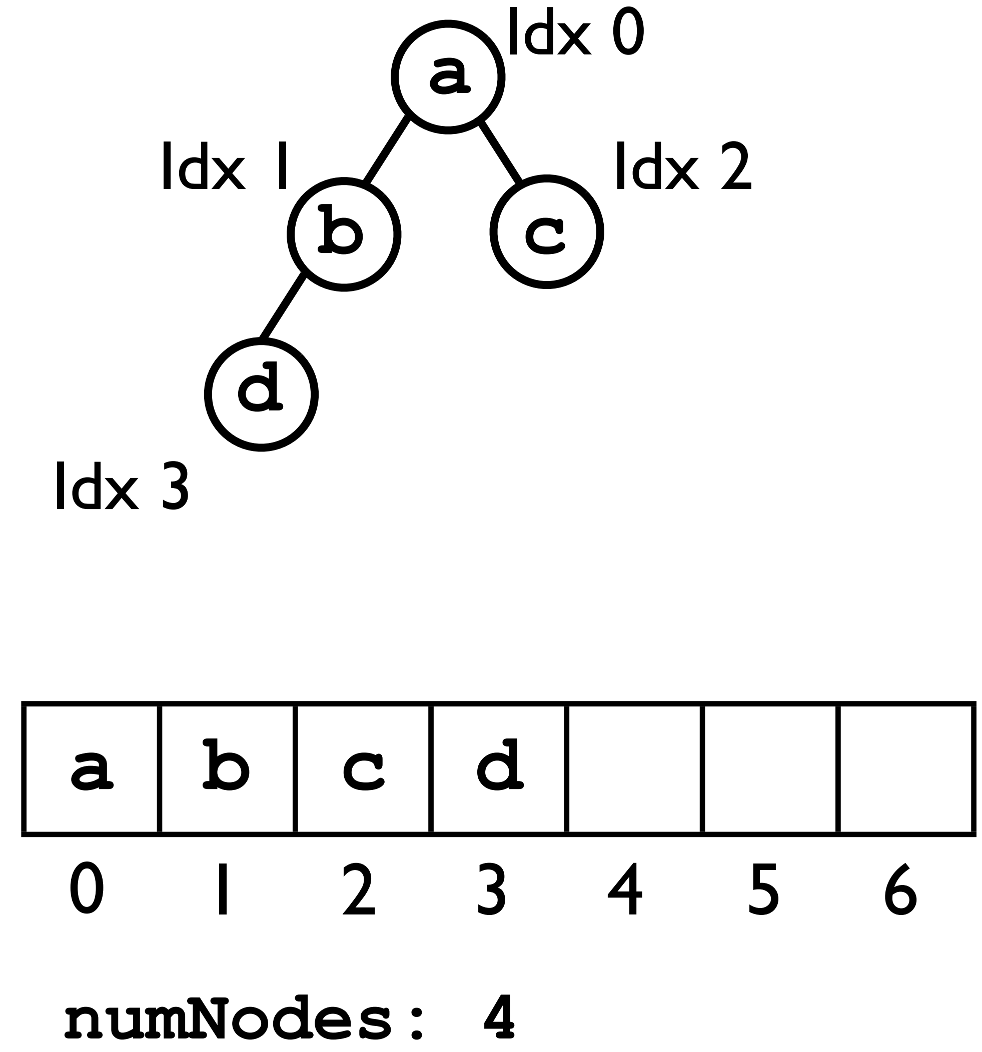
- Even though the data are being stored in a regular Java array, *their locations in the array still encode a tree structure among them*.
  - This means that binary tree-based algorithms we develop can still offer time-cost advantages over linear lists.



a	b	c	d	e	f	g
0	1	2	3	4	5	6

Adding a node

- Given that the binary tree must be *complete*, it is only valid to add a node *n* to be the *next child on the last level of the tree.*
- The index into the array of where this "next child" should be stored is always just **`_numNodes`**, where **`_numNodes`** is the current number of nodes in the tree.



Adding a node

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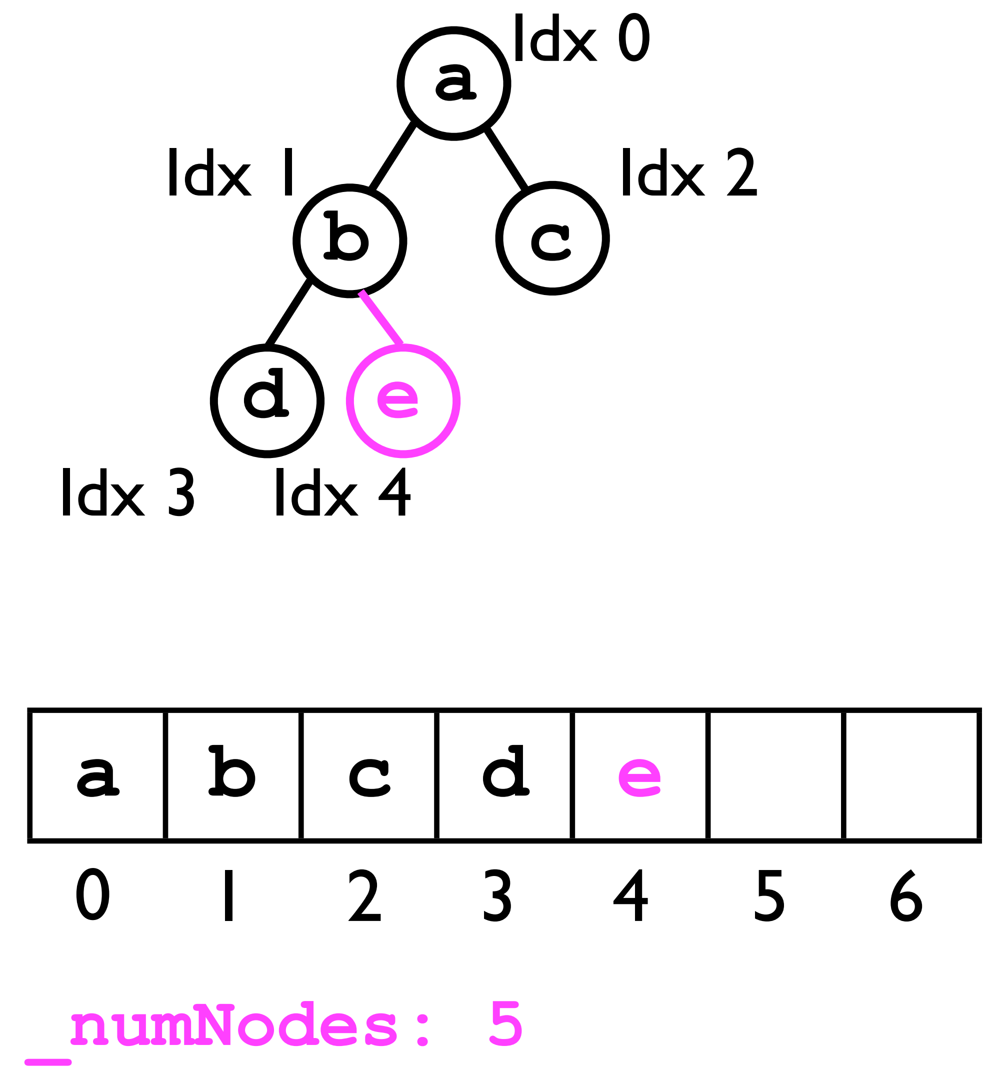
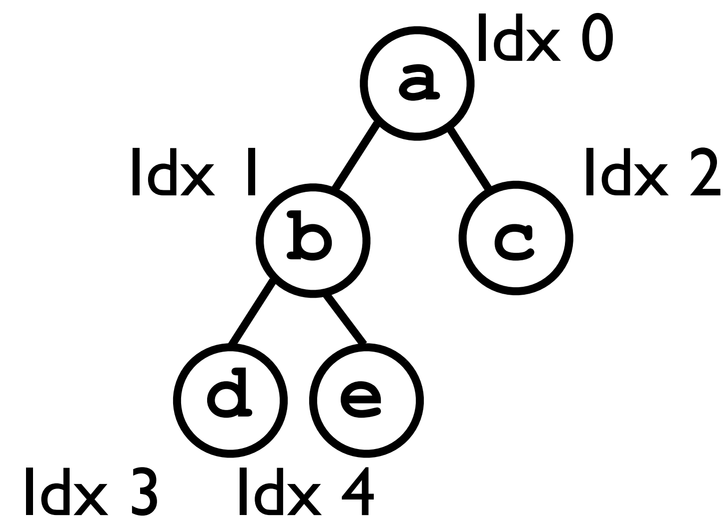
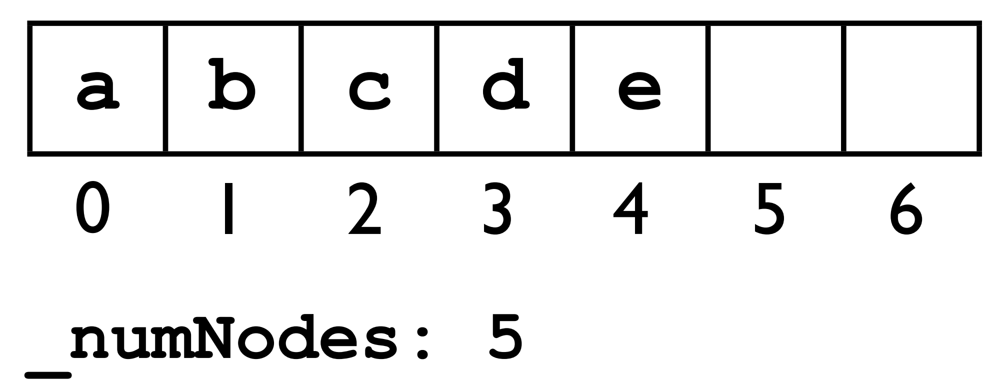


Removing a node

- Similarly, it is only valid to remove the right-most child of the last level of the tree.
- All we must do is decrement **`_numNodes`** to indicate that the "slot" in the array of the removed node is no longer valid.





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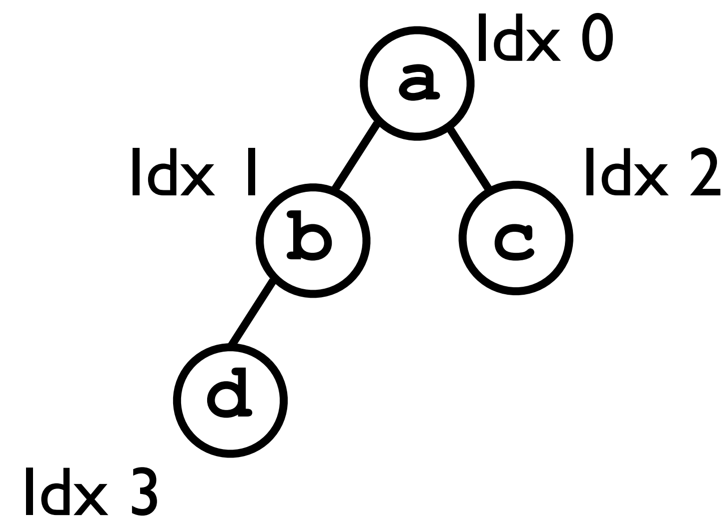


a	b	c	d			
0	1	2	3	4	5	6

`_numNodes: 4`

Finding a node

- To find the index of a node *n* whose data element equals o:

```
int find (int rootIdx, T o) {
  if (_nodeArray[rootIdx].equals(o)) {
    return rootIdx;
  }

  int idx;
  if (leftChild(rootIdx) < _numNodes &&
      (idx = find(leftChild(rootIdx), o)) >= 0) {
    return idx;
  } else if (rightChild(rootIdx) < _numNodes &&
      (idx = find(rightChild(rootIdx), o)) >= 0) {
    return idx;
  } else {
    return -1;
  }
}
```

Make sure each child exists before recursing!

Helper methods to determine index of left and right child nodes.

Binary trees to accelerate search.

Binary trees to accelerate search

- We have now constructed considerable "infrastructure" for building binary trees, using either "linked nodes" or a Java array for the tree's underlying storage.
- Trees are useful in their own right for representing *hierarchical structures*, e.g., genealogical data.
- However, for the moment we are interested in how they can *store* and *accelerate search* of data on which an *ordering relation* is defined.

Binary trees to accelerate search

- *Heaps* and *binary search trees* are two ADTs based on binary trees that accelerate search.
- A heap offers fast access to the largest element in a collection of related objects.
    - *O*(1) worst-case time cost for findLargest.
    - *O*(log *n*) worst-case time cost for removeLargest.
    - *O*(log *n*) worst-case time cost for add.
    - *O*(*n*) worst-case time-cost for find and remove.

Binary trees to accelerate search

- A binary search tree (BST) offers:
  - *O*(log *n*) average-case time cost for add, find, remove, and findLargest.
  - *O*(*n*) worst-case time cost for add, find, remove, and findLargest.
- AVL trees and red-black trees are more complicated, but they offer:
  - *O*(log *n*) worst-case time cost for add, find, remove, and findLargest.

Why findLargest?

- Why would we want to find the largest data element stored in a container?
- The **`findLargest`** method is required by *priority queues*.
  - A *priority queue* is a queue in which elements are dequeued not in FIFO order, but instead *in order of highest-to-lowest priority*.
  - A priority queue is typically implemented using a *heap*.

Taken from Paul Kube's slides.

Heaps.

Heaps

- A *max-heap* is an ADT for storing data so that the *largest element* (according to some binary order relation) can always be found and removed quickly.
- A *min-heap* is defined analogously for the *smallest element*.
- Internally, a *heap* is a *complete* binary tree which satisfies the *heap condition*:
  - The root of every sub-tree is *no smaller than any node in the sub-tree*. (For *max*-heap).
  - The root of every sub-tree is *no larger than any node in the sub-tree*. (For *min*-heap).
- This ensures that, to implement findLargest/findSmallest, *we can always just return the root node of the tree.*

Heaps

- A *max-heap* has the following interface:

```
// All operations must preserve the heap condition.
interface MaxHeap {
  // Adds o to the heap.
  void add (T o);
  // Removes the node whose data element equals o.
  void remove (T o);
  // Removes and returns the largest node in the heap.
  T removeLargest ();
  // Returns the largest node in the heap.
  T findLargest ();
  // Finds and returns the node whose data element
  // equals o.
  T find (T o);
  // Returns the number of data stored in the heap.
  int size ();
}
```

Implementing heaps

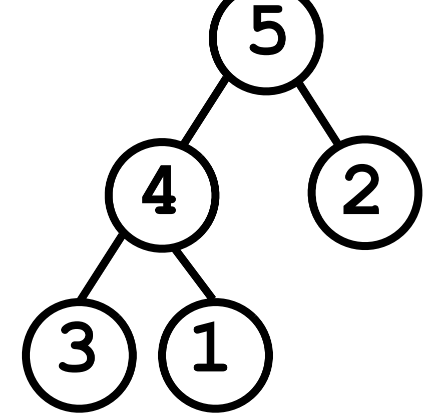
- Since heaps are anyway a *complete* binary tree, it is convenient and efficient to implement them using an array.
  - They can also be implemented using `Node` objects, but this is awkward.
- The challenge when implementing a heap is to preserve the heap property upon every *mutation* to the heap (add/remove).

Adding a node to a heap

- In order to add a new element o to a max-heap while *preserving the heap condition*, we execute the following procedure:
  - Add a new node *n* containing o to the last level of the tree (ensure *completeness* of the tree).
    - *This may violate the tree's heap condition* because o may be larger than one of its parents.
  - We then "fix" the heap by "swapping" node *n* with its parent *p*.
    - We repeat this process -- known as *bubbling* up -- as many times as necessary until the tree is a *heap* again.

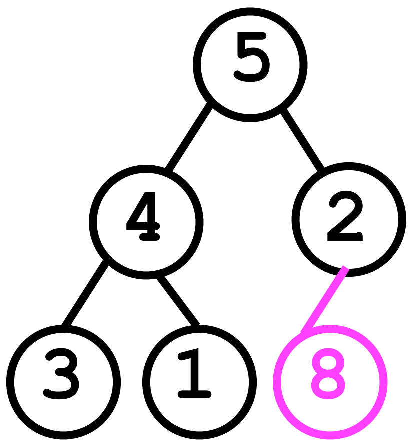
Adding a node to a heap

- Consider the max-heap to the right. (Notice that it satisfies the *heap condition*).



Adding a node to a heap

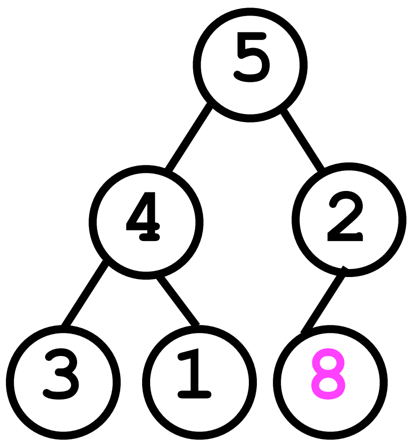
- Consider the max-heap to the right. (Notice that it satisfies the *heap condition*).
- Suppose we add value 8 to the bottom-level of the heap.
  - The tree no longer satisfies the heap condition.



2 is smaller than one of the nodes in its sub-tree!

Adding a node to a heap

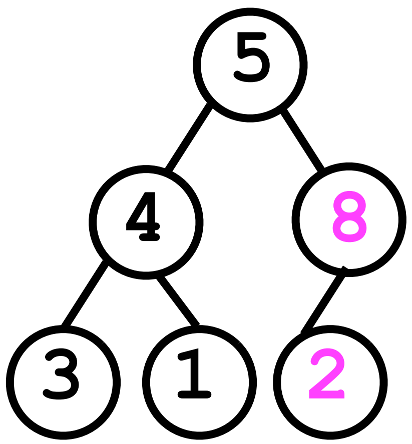
- Consider the max-heap to the right. (Notice that it satisfies the *heap condition*).



- Suppose we add value 8 to the bottom-level of the heap.
    - The tree no longer satisfies the heap condition.
    - We have to “bubble up” the 8 we just added to restore the heap condition.

Adding a node to a heap

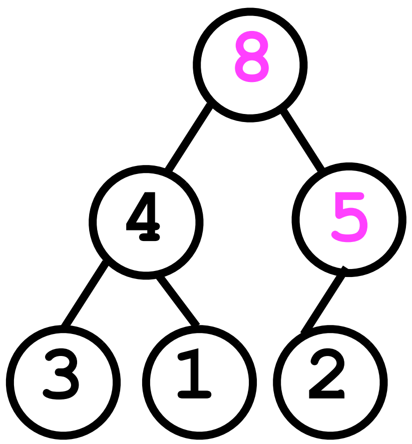
- Consider the max-heap to the right. (Notice that it satisfies the *heap condition*).
- Suppose we add value 8 to the bottom-level of the heap.
  - The tree no longer satisfies the heap condition.
  - We have to “bubble up” the 8 we just added to restore the heap condition.



Not done yet -- 5 is still smaller than 8.

Adding a node to a heap

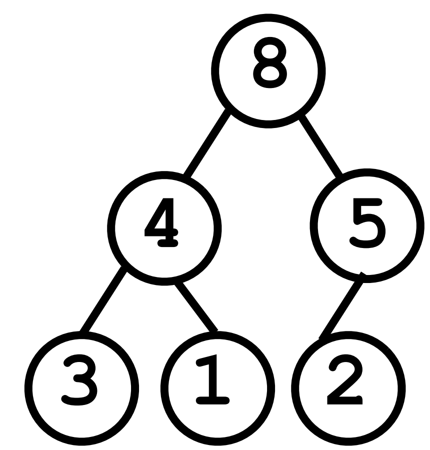
- Consider the max-heap to the right. (Notice that it satisfies the *heap condition*).
- Suppose we add value 8 to the bottom-level of the heap.
    - The tree no longer satisfies the heap condition.
    - We have to "bubble up" the 8 we just added to restore the heap condition.



Now it is a heap again!

Adding a node to a heap

- Consider the max-heap to the right. (Notice that it satisfies the *heap condition*).
- Suppose we add value 8 to the bottom-level of the heap.
  - The tree no longer satisfies the heap condition.
  - We have to "bubble up" the 8 we just added to restore the heap condition.
  - Done!



Adding a node to a heap

- We can implement the `add(o)` method as:

```
void add (T o) {
  _nodeArray[_numNodes] = o;
  _numNodes++;
  bubbleUp(_numNodes - 1);
}
```

- We must then also implement `bubbleUp(idx)`:

```
void bubbleUp (int idx) {
  If node at idx is "larger" than its parent:
    Swap data in the node and its parent;
    Recursively bubbleUp(parentIdx(idx));
}
```

Adding a node to a heap

- Alternatively, we can write an *iterative* version of `bubbleUp(idx)`:

```
void bubbleUp (int idx) {
  While node at idx is “larger” than its parent:
    Swap data in the node and its parent;
    Set idx to be parentIdx(idx);
}
```

Next lecture

- Finding and removing elements.
- “Trickling down” a node.