CSE 12: Basic data structures and object-oriented design

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Lecture Nineteen
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Quicksort.

Quicksort

- The last sorting algorithm we consider is **Quicksort**.
- Quicksort has one of the best performance profiles of all the general-purpose sorting algorithms in the *average case*.
- Like Mergesort, Quicksort is based on the *divide-and-conquer* principle.
- Quicksort differs from Mergesort in how it divides the input array into two pieces.

Quicksort

- The high-level idea of Quicksort is the following:
  - Rearrange ("partition") the input array into a *left part L* and a *right part R* so that:

    everything in the left part $\leq$ everything in the right part.
  - Then, recursively call Quicksort on both the left and right halves.

Quicksort

- Pseudocode:

```
void quicksort (array) {
  If array.length == 1, then do nothing.
  Else:
    Partition array into left part and right part, so that:
      everything in left part ≤ everything in right part.
    quicksort(leftPart);
    quicksort(rightPart);
}
```

Partitioning

- The key to Quicksort is the `partition` function, which needs to operate in $O(n)$ time.
- `partition(array)` works by picking a *pivot* element x from `array`.
  - Left part contains elements $\leq x$.
  - Right part contains elements $\geq x$.
- The simplest implementations choose the *first* element of `array` as the pivot.
- Better-performing implementations choose a *random* element of `array`.

Partitioning

- The `partition` method will work as follows:

```
void partition (array) {
  pivot = pickPivot(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

- This procedure will effectively move all elements ≤ `pivot` to the *left*, and all elements ≥ `pivot` to the *right*.

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

6 1 4 3 8 7 2 5

☝ ☝

i = -1 **pivot = 6** **j = 8**

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

```
6 1 4 3 8 7 2 5
☝                           ☝

i = 0      pivot = 6      j = 8
```

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

6 1 4 3 8 7 2 5

☝ ☝

i = 0 **pivot = 6** **j = 7**

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

5 1 4 3 8 7 2 6

☝ ☝

```
i = 0        pivot = 6        j = 7
```

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let’s try an example where we select the pivot to just be the array’s *first element*:

5 1 4 3 8 7 2 6

i = 1 **pivot = 6** **j = 7**

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

```
5 1 4 3 8 7 2 6
```

i = 2   pivot = 6   j = 7

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

5 1 4 3 8 7 2 6

i = 3   **pivot = 6**   **j = 7**

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

```
5 1 4 3 8 7 2 6
        ☝     ☝

i = 4      pivot = 6      j = 7
```

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

5 1 4 3 8 7 2 6

☝ ☝

i = 4 **pivot = 6** **j = 6**

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

```
5 1 4 3 2 7 8 6
        ☝   ☝

i = 4      pivot = 6      j = 6
```

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let’s try an example where we select the pivot to just be the array’s *first element*:

5 1 4 3 2 7 8 6

☝ ☝

`i = 5` `pivot = 6` `j = 6`

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

5 1 4 3 2 7 8 6

i = 5 **pivot = 6** **j = 5**

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

5 1 4 3 2 7 8 6

i = 5 pivot = 6 j = 4

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

5 1 4 3 2 7 8 6

☝ ☝

i = 5 **pivot = 6** **j = 4**

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

5 1 4 3 2 7 8 6

☝ ☝

i = 5 pivot = 6 j = 4

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

Partitioning

- Let's try an example where we select the pivot to just be the array's *first element*:

Left part Right part

5 1 4 3 2 7 8 6

☝ ☝

```
i = 5        pivot = 6        j = 4
```

Done.

```
void partition (array) {
  pivot = pickRandomElement(array);
  Set i = -1
  Set j = N
  while i < j:
    Increment i until array[i] ≥ pivot.
    Decrement j until array[j] ≤ pivot.
    If i < j, then swap array[i] and array[j].
}
```

The partition method also has the nice side effect that the final value of `j` tells us the *right-most* element in the *left part.*

Quicksort

- Example:

`6 1 4 3 8 7 2 5`

Partition.

Quicksort

- Example:

```
6 1 4 3 8 7 2 5
5 1 4 3 2 7 8 6
```

Recurse.

Left part Right part

Quicksort

- Example:

```
6 1 4 3 8 7 2 5
5 1 4 3 2 7 8 6

5 1 4 3 2           7 8 6
```

Partition.

Quicksort

- Example:

6 1 4 3 8 7 2 5

5 1 4 3 2 7 8 6

5 1 4 3 2 7 8 6

2 1 4 3 5 6 8 7

Recurse.

Left part Right part Left part Right part

Quicksort

- Example:

6 1 4 3 8 7 2 5

5 1 4 3 2 7 8 6

5 1 4 3 2 7 8 6

2 1 4 3 5 6 8 7

2 1 4 3 5 6 8 7 Partition.

Quicksort

- Example:

```
6 1 4 3 8 7 2 5
5 1 4 3 2 7 8 6

5 1 4 3 2           7 8 6
2 1 4 3 5           6 8 7

2 1 4 3     5     6     8 7
1 2 4 3     5     6     7 8
```

Recurse.

Quicksort

- Example:

```
        6 1 4 3 8 7 2 5
        5 1 4 3 2 7 8 6

   5 1 4 3 2         7 8 6
   2 1 4 3 5         6 8 7

  2 1 4 3    5    6     8 7
  1 2 4 3    5    6     7 8

1     2 4 3  5    6   7   8
```

Partition.

Quicksort

- Example:

```
            6 1 4 3 8 7 2 5
            5 1 4 3 2 7 8 6

   5 1 4 3 2            7 8 6
   2 1 4 3 5            6 8 7

  2 1 4 3     5     6      8 7
  1 2 4 3     5     6      7 8

1     2 4 3   5     6   7    8
1     2 4 3   5     6   7    8
```

Recurse.

Quicksort

- Example:

```
            6 1 4 3 8 7 2 5
            5 1 4 3 2 7 8 6

   5 1 4 3 2          7 8 6
   2 1 4 3 5          6 8 7

  2 1 4 3    5     6     8 7
  1 2 4 3    5     6     7 8

1     2 4 3  5     6   7   8
1     2 4 3  5     6   7   8

1   2   4 3  5     6   7   8
```

Partition.

Quicksort

- Example:

6 1 4 3 8 7 2 5
5 1 4 3 2 7 8 6

5 1 4 3 2 7 8 6
2 1 4 3 5 6 8 7

2 1 4 3 5 6 8 7
1 2 4 3 5 6 7 8

1 2 4 3 5 6 7 8
1 2 4 3 5 6 7 8

1 2 4 3 5 6 7 8
1 2 3 4 5 6 7 8

Recurse.

Quicksort

- Example:

```
        6 1 4 3 8 7 2 5
        5 1 4 3 2 7 8 6

   5 1 4 3 2          7 8 6
   2 1 4 3 5          6 8 7

  2 1 4 3    5     6     8 7
  1 2 4 3    5     6     7 8

1    2 4 3   5     6   7   8
1    2 4 3   5     6   7   8

1  2   4 3   5     6   7   8
1  2   3 4   5     6   7   8
1  2  3  4   5     6   7   8      Done.
```

Quicksort

- We can also do this *in-place*:

`6 1 4 3 8 7 2 5`

Done.

Quicksort

- We can also do this *in-place*:

```
6 1 4 3 8 7 2 5
5 1 4 3 2 7 8 6
```

Done.

Quicksort

- We can also do this *in-place*:

```
6 1 4 3 8 7 2 5
5 1 4 3 2 7 8 6
2 1 4 3 5 6 8 7
```

Done.

Quicksort

- We can also do this *in-place*:

```
6 1 4 3 8 7 2 5
5 1 4 3 2 7 8 6
2 1 4 3 5 6 8 7
1 2 4 3 5 6 7 8
```

Done.

Quicksort

- We can also do this *in-place*:

```
6 1 4 3 8 7 2 5
5 1 4 3 2 7 8 6
2 1 4 3 5 6 8 7
1 2 4 3 5 6 7 8
1 2 4 3 5 6 7 8
```

Done.

Quicksort

- We can also do this *in-place*:

```
6 1 4 3 8 7 2 5
5 1 4 3 2 7 8 6
2 1 4 3 5 6 8 7
1 2 4 3 5 6 7 8
1 2 4 3 5 6 7 8
1 2 3 4 5 6 7 8
```

Done.

Quicksort

- We can also do this *in-place*:

```
6 1 4 3 8 7 2 5
5 1 4 3 2 7 8 6
2 1 4 3 5 6 8 7
1 2 4 3 5 6 7 8
1 2 4 3 5 6 7 8
1 2 3 4 5 6 7 8
```

Done.

Quicksort

- The version of Quicksort just demonstrated operates *in-place*, but it is not *stable*.
    - Alternative implementations are *stable*, but do not operate in-place.

Quicksort

- With Quicksort, *all* the sorting all happens "on the way down" the stack of recursive calls.
  - As soon as every call to Quicksort has reached the base case, the array is *sorted*.
- Contrast this with Mergesort, in which the *merging* takes place "on the way back up" the stack of recursive calls.
  - As soon as every call to Mergesort has reached the base case, not even a single element has been re-arranged.

Mergesort

- Example: First stage: recursively divide until we reach the base case.

6 1 4 3 8 7 2 5

Split list and recurse.

```
void mergesort (array) {
  If array.length == 1, then do nothing.
  Else:
    Split array evenly into leftArray and rightArray.
    mergesort(leftArray);
    mergesort(rightArray);
    Merge the leftArray and rightArray into array
}
```

Mergesort

- Example: First stage: recursively divide until we reach the base case.

6 1 4 3 8 7 2 5

6 1 4 3 **8 7 2 5**

Split list and recurse.

```
void mergesort (array) {
  If array.length == 1, then do nothing.
  Else:
    Split array evenly into leftArray and rightArray.
    mergesort(leftArray);
    mergesort(rightArray);
    Merge the leftArray and rightArray into array
}
```

Mergesort

- Example: First stage: recursively divide until we reach the base case.

```
6 1 4 3 8 7 2 5                   Split list and recurse.

6 1 4 3         8 7 2 5           Split list and recurse.

6 1     4 3     8 7     2 5       Split list and recurse.

6   1   4   3   8   7   2   5
```

```
void mergesort (array) {
  If array.length == 1, then do nothing.
  Else:
    Split array evenly into leftArray and rightArray.
    mergesort(leftArray);
    mergesort(rightArray);
    Merge the leftArray and rightArray into array
}
```

Mergesort

- Example:

Each of these is a "list" (size 1) passed to a recursive call to Mergesort.

6 1 4 3 8 7 2 5

```
void mergesort (array) {
  If array.length == 1, then do nothing.
  Else:
    Split array evenly into leftArray and rightArray.
    mergesort(leftArray);
    mergesort(rightArray);
    Merge the leftArray and rightArray into array
}
```

Mergesort

- Example:

Second stage: merge each pair of sorted sub-lists.

1 6 3 4 7 8 2 5

Merge the two sub-lists.

6 1 4 3 8 7 2 5

```
void mergesort (array) {
  If array.length == 1, then do nothing.
  Else:
    Split array evenly into leftArray and rightArray.
    mergesort(leftArray);
    mergesort(rightArray);
    Merge the leftArray and rightArray into array
}
```

Mergesort

- Example: Second stage: merge each pair of sorted sub-lists.

1 3 4 6 **2 5 7 8** Merge the two sub-lists.

1 6 **3 4** **7 8** **2 5**

6 1 4 3 8 7 2 5

```
void mergesort (array) {
  If array.length == 1, then do nothing.
  Else:
    Split array evenly into leftArray and rightArray.
    mergesort(leftArray);
    mergesort(rightArray);
    Merge the leftArray and rightArray into array
}
```

Mergesort

- Example:

Second stage: merge each pair of sorted sub-lists.

Merge the two sub-lists.

1 2 3 4 5 6 7 8

1 3 4 6 2 5 7 8

1 6 3 4 7 8 2 5

6 1 4 3 8 7 2 5

```
void mergesort (array) {
  If array.length == 1, then do nothing.
  Else:
    Split array evenly into leftArray and rightArray.
    mergesort(leftArray);
    mergesort(rightArray);
    Merge the leftArray and rightArray into array
}
```

Mergesort

- Example:

Done.

1 2 3 4 5 6 7 8

1 3 4 6　　　　2 5 7 8

1 6　　3 4　　7 8　　2 5

6　1　4　3　8　7　2　5

```
void mergesort (array) {
  If array.length == 1, then do nothing.
  Else:
    Split array evenly into leftArray and rightArray.
    mergesort(leftArray);
    mergesort(rightArray);
    Merge the leftArray and rightArray into array
}
```

Quicksort

- The time cost of Quicksort in the *average case* differs substantially from the *worst case*.
- In the average case, the `partition` procedure splits the array into equal-sized parts.
  - This results in a recursion depth of $O(\log n)$.
- At each level of recursion, the entire array must be "touched" (during partitioning), so n.
- In total, Quicksort is $n * O(\log n) = O(n \log n)$.

Quicksort

- In the *worst case*, the `partition` procedure splits the array into a 1-element part, and a n-1-element part.
- If this occurs throughout the entire recursion stack, then the recursion depth will be $O(n)$ instead of $O(\log n)$.
- Since every element must still be "touched" at each level of recursion, this results in $O(n) * O(n) = O(n^2)$ operations.
    - Hence, in the worst case, Quicksort is no better than insertion/selection sort.

Quicksort

- This worst case is realized if (a) the input array is already sorted and (b) we always choose the first element to be the pivot.
- Example:

Left part Right part

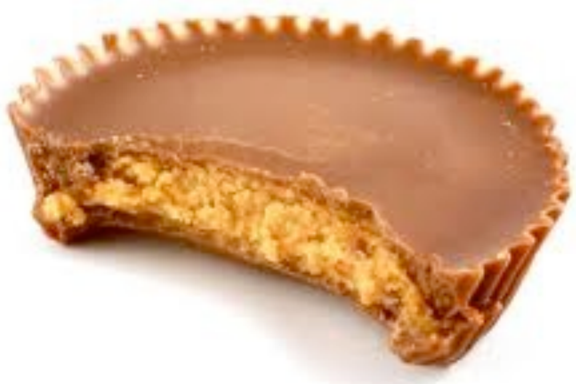
```
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
```

n levels deep

Quicksort

- To prevent this worst-case performance from happening, practical implementations of Quicksort pick the pivot element *randomly*.
  - This ensures that, on a list that is already sorted, Quicksort still gives $O(n \log n)$ performance.

Objects can be expensive.



This is the last slide of the course.