CSE 12:
Basic data structures and object-oriented design

Jacob Whitehill
jake@mplab.ucsd.edu

Lecture Fifteen
25 Aug 2011

Hash tables.

Recap of BSTs

- Binary search trees (BSTs) offer *O*(log *n*) time costs for **`add`**/**`remove`**/**`find`** operations by exploiting order relationships among data.
- They are also memory efficient, in that only as many nodes are allocated as elements are contained in the BST.
- However, by utilizing more *memory* (greater space complexity), we can achieve *even higher* performance (lower time complexity).
- **Hash sets** and **hash maps** offer *O*(1) time costs for **`add`**/**`remove`**/**`find`** operations by investing more memory in the underlying storage.

Finite "universe" of objects to store

- Suppose you were developing a database of UCSD students.
- You may wish to build a function that allows you to retrieve a student's full record given their student ID:

```
class Student {
  int _studentID;  // 4 bytes long
  String _firstName, _lastName;
  String _address;
  String _favoriteColor;
}
```

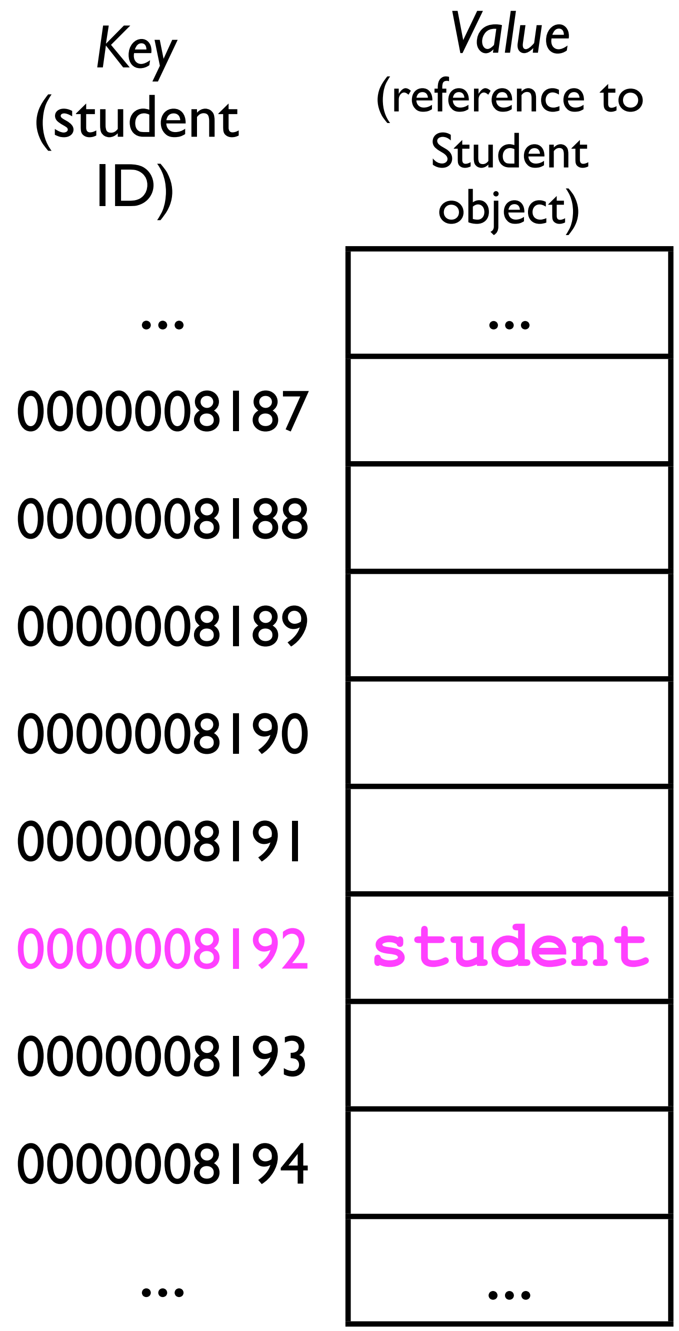
- In this application, the *key* would be the student ID, and the *value* associated with the key would be the whole **`Student`** record.

Finite “universe” of objects to store

- In this application (and others), there may exist only a finite **universe** of possible keys values.
    - For instance, a 4-byte integer can only store about 4 billion different values ==> 4 billion unique keys.
- Sometimes this finite set of keys is small enough that we can allocate an array big enough to give *every possible key its own slot*.
- In this case, we can make the “search” process for a particular key *trivial*:
    - We simply “jump” to the unique array index assigned to that key.
    - This takes only *O*(1) time in the *worst*-case.

Finite "universe" of objects to store

- For example, in a UCSD student database application, we could allocate an array 4 billion elements long.
- Whenever we wished to find a particular student, we simply *jump* to that student's correct slot using the student ID as the *index*.
  - E.g., student with ID# 0000008192 would be stored at slot 0000008192.
  - Search is *trivial* -- *O*(1).



Key (student ID)	*Value* (reference to Student object)
...	...
0000008187	
0000008188	
0000008189	
0000008190	
0000008191	
0000008192	`student`
0000008193	
0000008194	
...	...

Finite “universe” of objects to store

- Similarly, when adding a student, we simply insert an entry at his/her *unique location* in the array:

```
final Student other =
  new Student(8188, ...);
_container.add(other);
```

- Since student IDs are guaranteed to be unique, there is exactly *one* element that will ever reside at the index of the array.

Key (student ID)	*Value* (reference to Student object)
...	
0000008187	
0000008188	`other`
0000008189	
0000008190	
0000008191	
0000008192	`student`
0000008193	
0000008194	
...	...

Finite “universe” of objects to store

- Using the idea sketched above, we could implement a `StudentDatabase` with excellent time costs.

```
class StudentDatabase {
  Student[] _allPossibleStudents = new Student[4294967296];

  void add (Student s) {  // O(1)
    final int index = s._studentID;
    _allPossibleStudents[index] = s;
  }

  // Returns null if s is not in the database
  Student get (Student s) {  // O(1)
    final int index = s._studentID;
    return _allPossibleStudents[index];
  }

  void remove (Student s) {  // O(1)
    final int index = s._studentID;
    _allPossibleStudents[index] = null;
  }
}
```

Finite “universe” of objects to store

- However, the space costs of this data structure are enormous.

```
class StudentDatabase {
  Student[] _allPossibleStudents = new Student[4294967296];

  void add (Student s) {  // O(1)
    final int index = s._studentID;
    _allPossibleStudents[index] = s;
  }

  // Returns null if s is not in the database
  Student get (Student s) {  // O(1)
    final int index = s._studentID;
    return _allPossibleStudents[index];
  }

  void remove (Student s) {  // O(1)
    final int index = s._studentID;
    _allPossibleStudents[index] = null;
  }
}
```

Finite "universe" of objects to store

- Allocating 4 billion entries for a UCSD student database is extremely wasteful.
  - The universe of keys is far larger than the number we expect to ever want to store.
- What if we decreased the size of the table from 4 billion entries to storing, say, just 100,000?
  - 100,000 is still far higher than the actual number of UCSD students, so there should be plenty of space.

Finite "universe" of objects to store

- With an array of only 100,000 entries, each student ID would *no longer have its own unique array index* -- multiple student IDs would have to "share" an index.
  - We call the "sharing" of an array index by 2 (or more) student IDs a **collision**.
  - Whenever a collision occurs, we have to store the **`Student`** object "somewhere else" (more later).
  - However, if we're clever about how we assign array indices to student IDs, then collisions will rarely occur.
  - We can still achieve *O*(1) add/find/remove time in the *average case*.

Hash tables

- This idea is called a "hash table."
- A **hash table** consists of a large array of *M* "slots" (or "buckets") to store the user's data.
- A hash table also requires:

  1. Some way of converting from an object's *key* into an *index* that specifies where that object should be stored.
     - This is called a *hash function*.
  2. A method of handling *collisions*.

- In order to ensure good performance, *M* must be bigger than *N*, the number of data the user will want to store.

Hash function

- A *hash function* maps an object's *key* into an array index, i.e., a number from 0...*M*-1, where *M* is the number of entries in the hash table.

- Simple example:

```
int hashFunction (int studentID) {
  return studentID % M;
}
```

- The *modulus* operator `%` divides `studentID` by `M`, and then returns the *remainder*. Examples:

  3 % 10 = 3
  107 % 10 = 7
  7 % 4 = 3
  16 % 5 = 1

Hash function

- To be useful, a hash function must be *fast*.
    - Its performance should not depend on the particular key.
- A hash function must also be *deterministic*:
    - Given the *same key*, it must *always* return the *same array index*.
        - (Otherwise, how would we find something we stored earlier?)

Hash function

- A “good” hash function should also be *uniform*:
  - Each “slot” *i* in the array should be equally likely to be chosen as any other slot *j*.
  - We certainly don’t want to map every possible key to the same array index!
  - Uniform is important to ensure good performance.

Hash function

- For instance, if **`M`** is 100000, then **`studentID % 100000`** is simply the *last 5 digits* of the student ID, e.g.:
    - **`student1`** with Student ID `0000013012` would map to index `13012`.
    - **`student2`** with Student ID `1234567890` would map to index `67890`.
    - These *indices* specify *where* in the array the students are *stored*.

Key (student ID)	*Value* (reference to Student object)
...	
13011	
13012	`student1`
13013	
...	
67889	
67890	`student2`
67891	
67892	
...	...

Handling collisions

- Unfortunately, on occasion, there would be two (or more) **`Student`** objects who are "hashed" (mapped) into the same array slot.

```
studentID1 = 2200012345;
studentID2 = 1926112345;
hashTable.add(studentID1, student1);
hashTable.add(studentID2, student2);
```

- This is called a *collision* -- two different **`Student`** objects map into the *same array index*.
- How do we handle these collisions?

Handling collisions

- There are two principal ways of handling collisions:

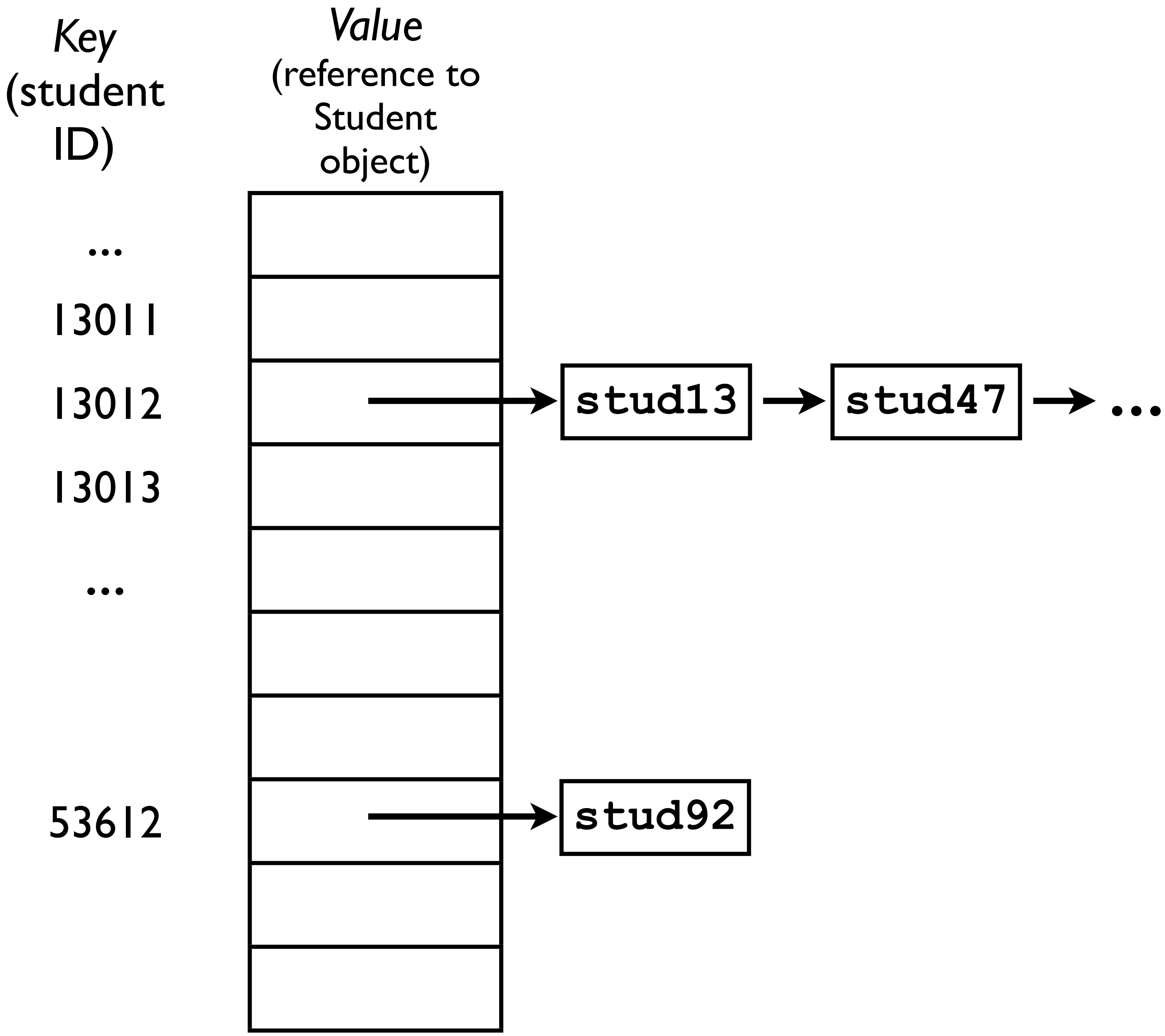
1. **Chaining** (aka **separate chaining**) -- at each slot in the array, instead of storing only a single element, we store a linked list of elements.

2. **Open addressing** -- if `student5` "hashes" to array index 123, and array index 123 is already occupied, then we look for "another" index at which to store `student5`, e.g., 124.

    - Different schemes for determining "another index".

Chaining

- Each slot in the array contains not an object itself, but rather a pointer to the *head* of a *linked list* of objects which all map to the same index.

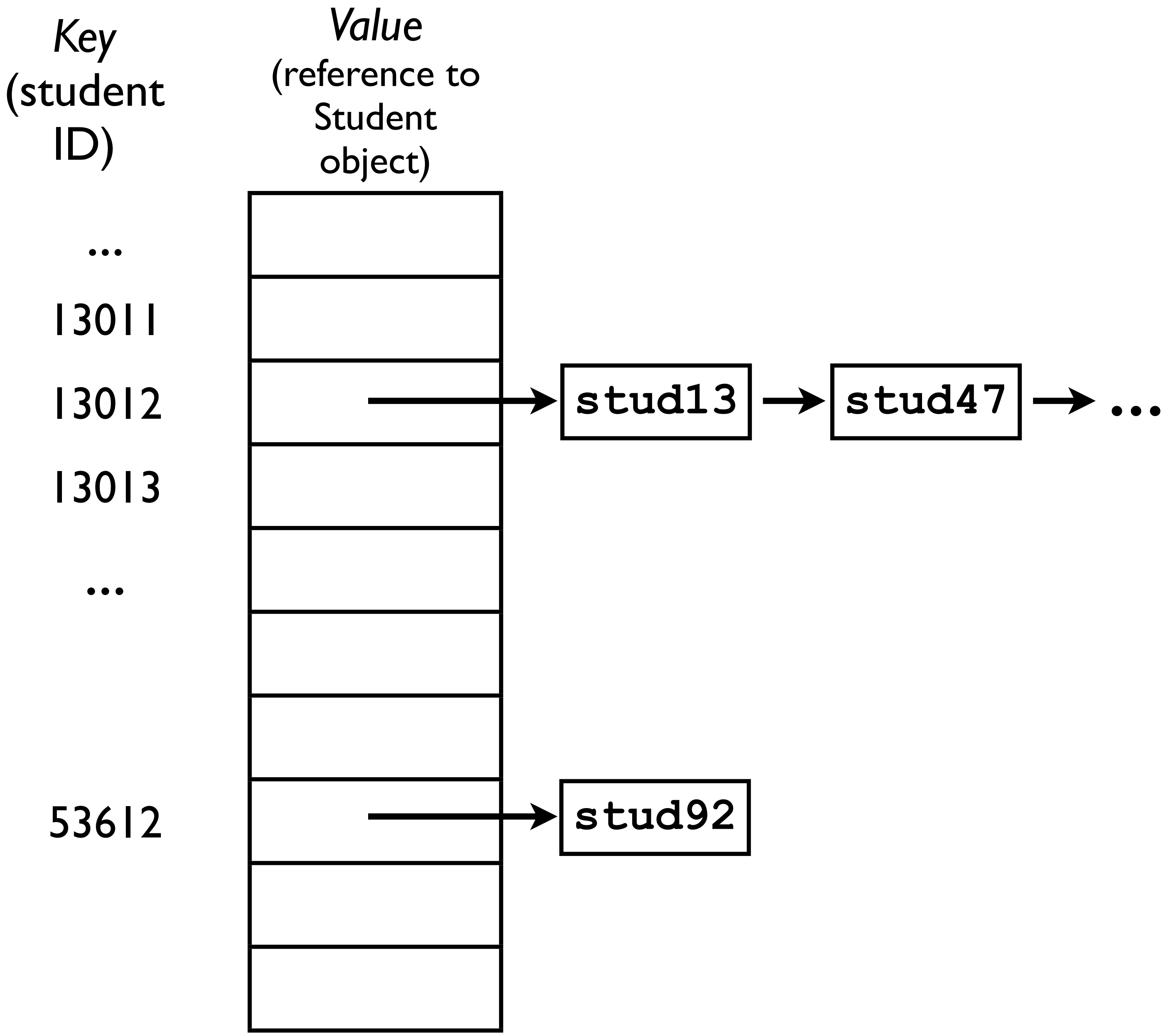


Chaining

- When looking for a particular object, we must:

  1. *Hash* the key to obtain the *index*.
  2. *Search* the list for the correct object.

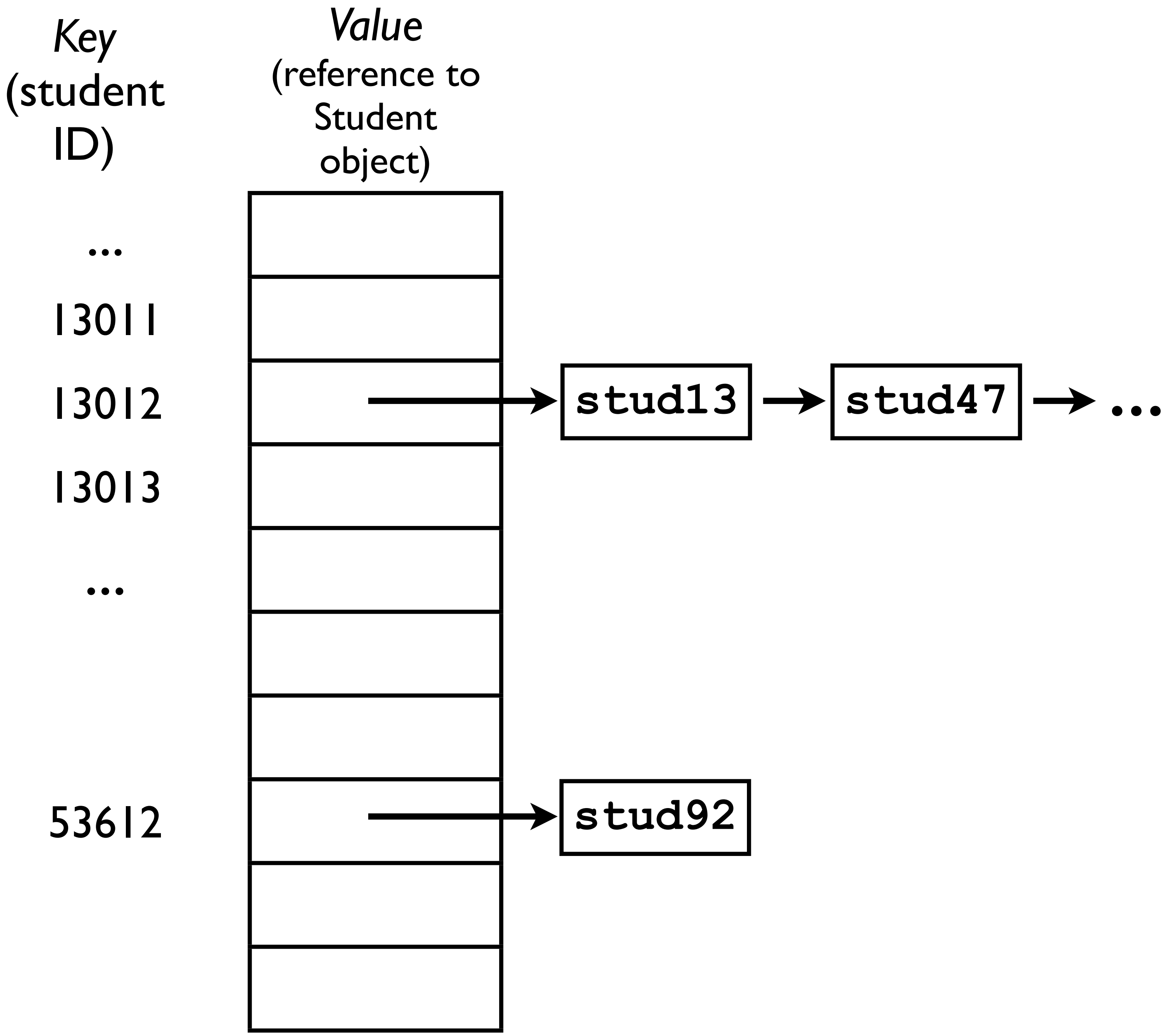
- This will still be *fast* as long as the linked lists are *short* (more later).



Chaining

- For example, if we wish to find `stud47` with student ID `0925113012`.

  1. Hash `stud47`'s student ID to determine the *index*.
  2. Jump to the head of the corresponding linked list.
  3. Traverse the linked list until we find `stud47`.



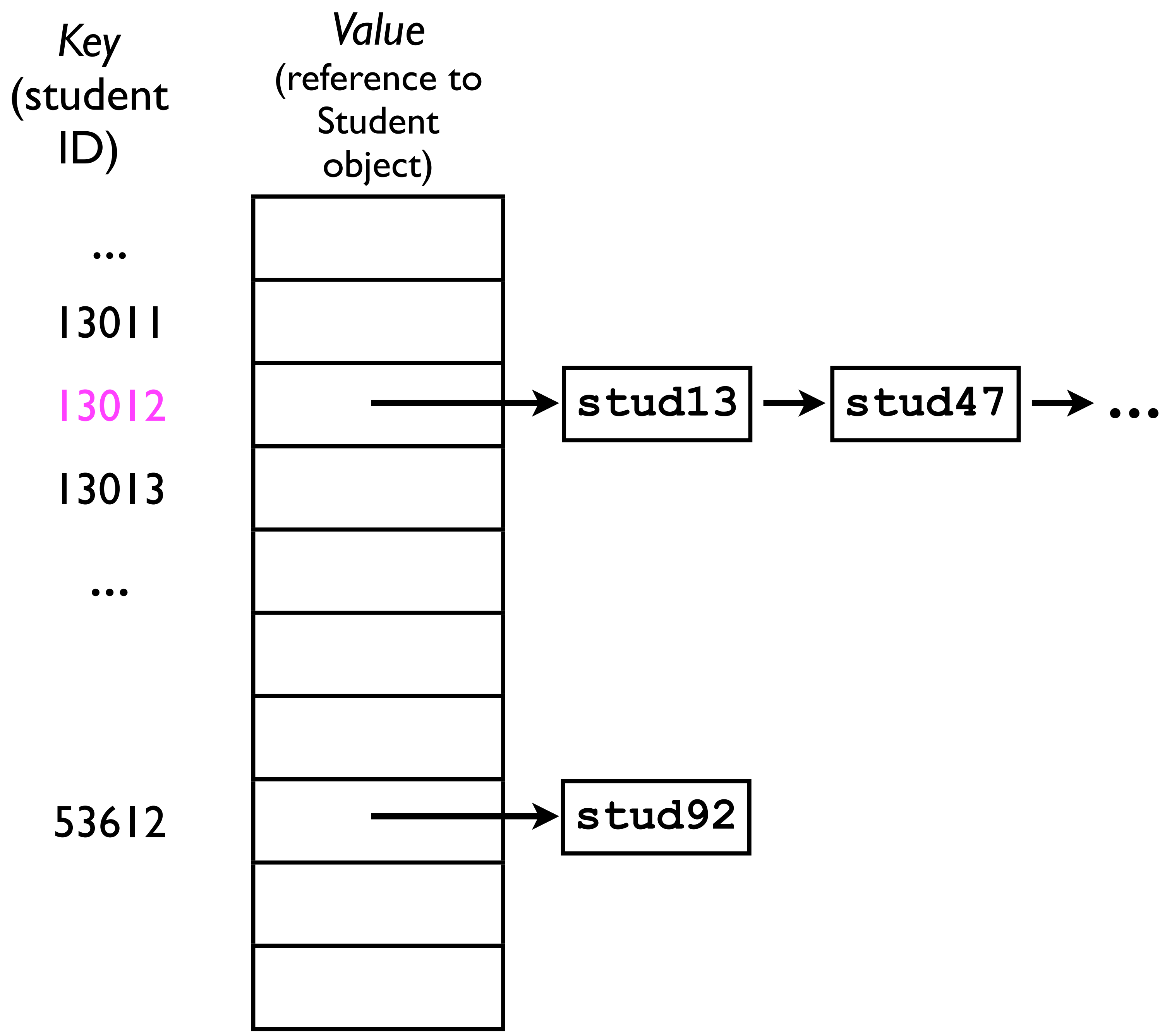
Chaining

- For example, if we wish to find `stud47` with student ID `0925113012`.

  1. Hash `stud47`'s student ID to determine the *index*.

  2. Jump to the head of the corresponding linked list.

  3. Traverse the linked list until we find `stud47`.



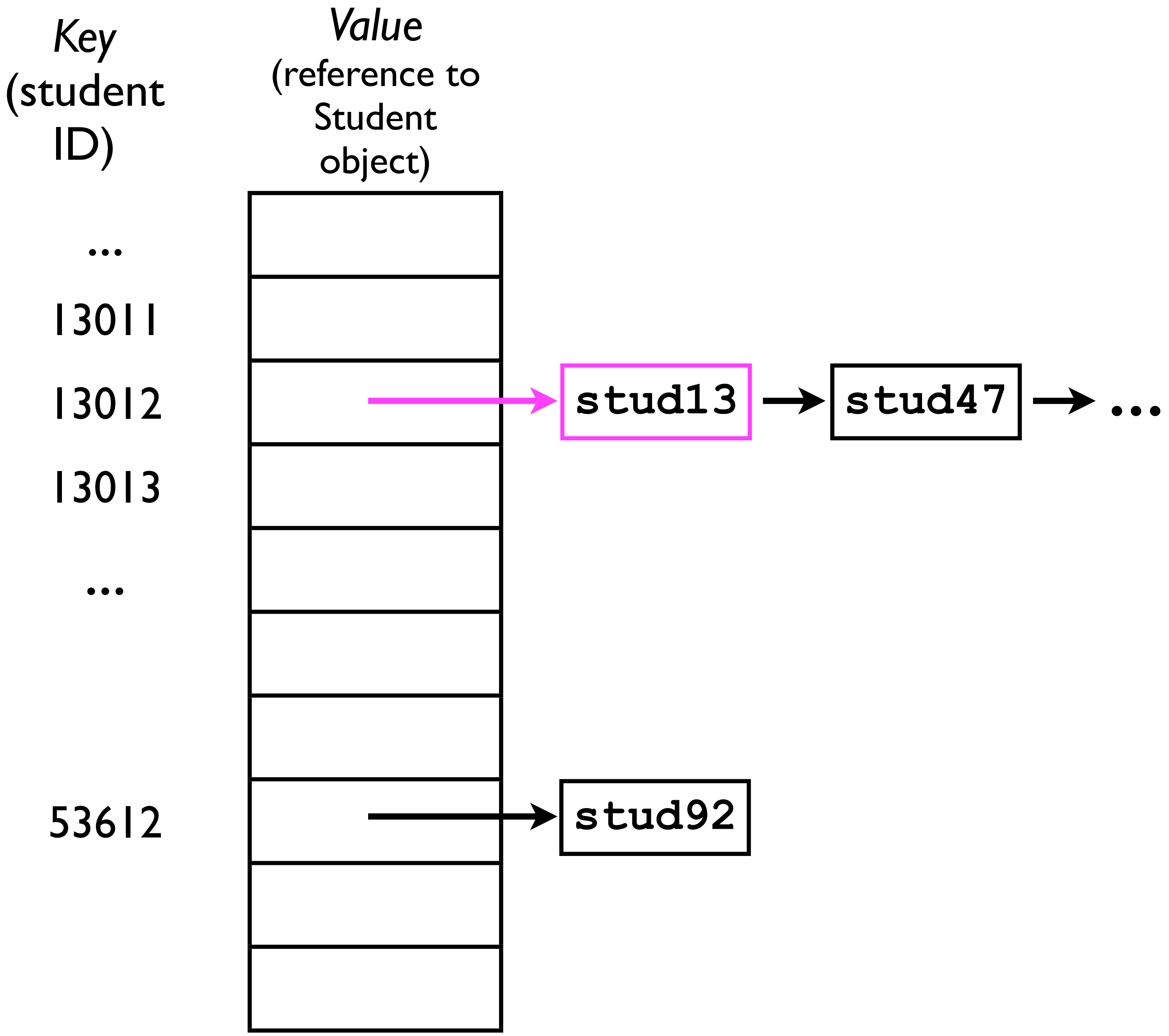
Chaining

- For example, if we wish to find **`stud47`** with student ID **`0925113012`**.

1. Hash **`stud47`**'s student ID to determine the *index*.

2. Jump to the head of the corresponding linked list.

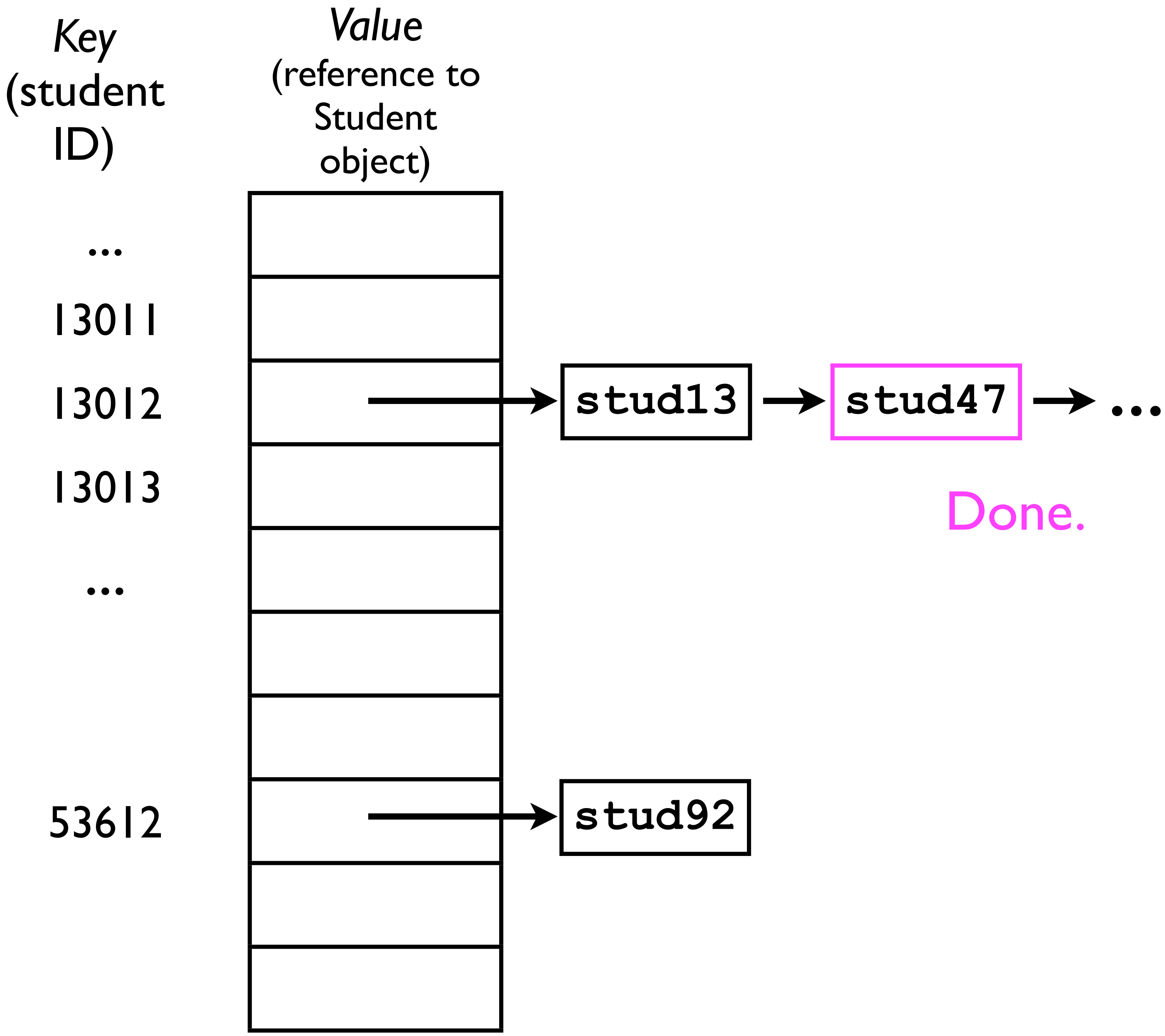
3. Traverse the linked list until we find **`stud47`**.



Chaining

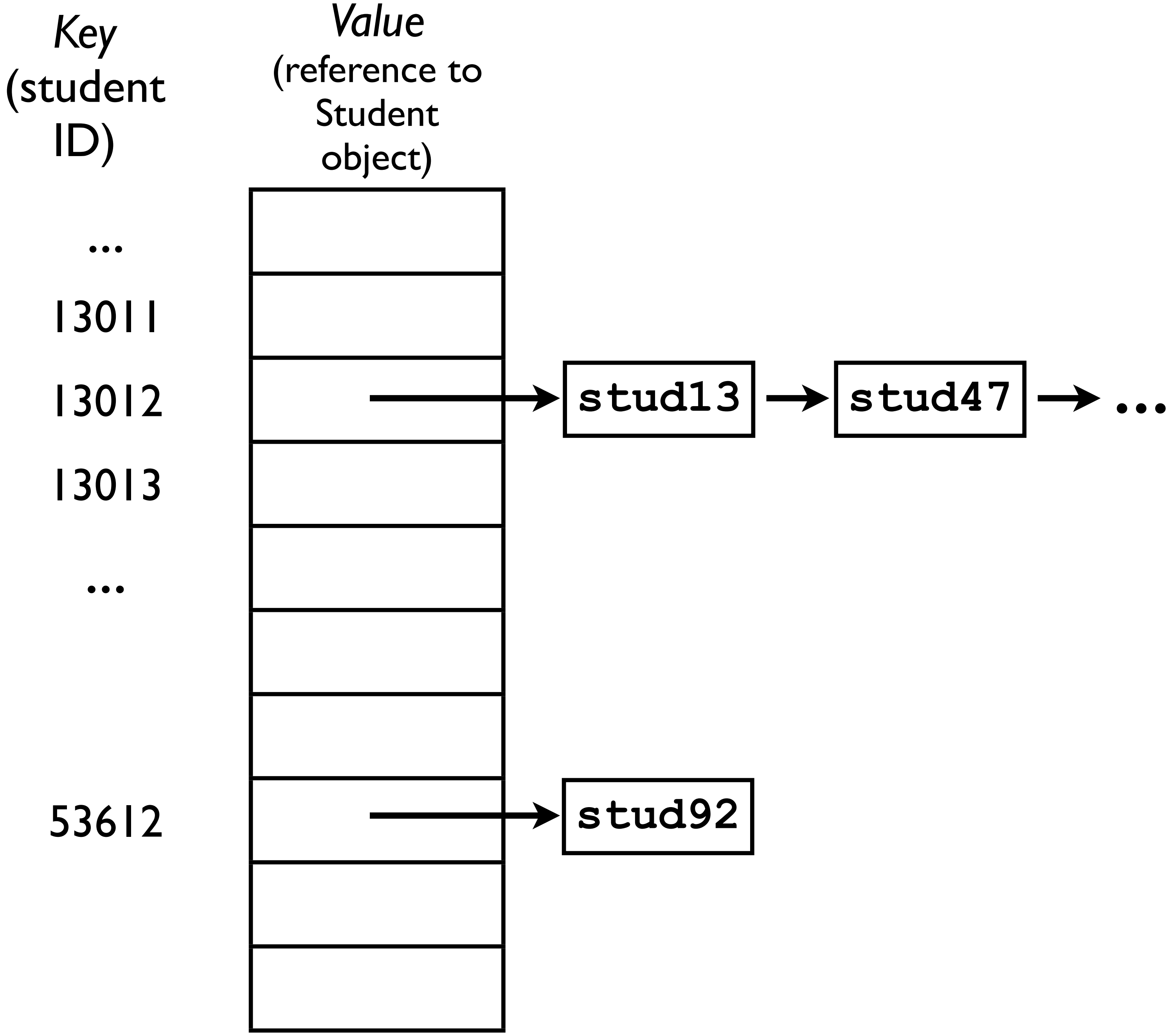
- For example, if we wish to find **`stud47`** with student ID **`0925113012`**.

1. Hash **`stud47`**'s student ID to determine the *index*.
2. Jump to the head of the corresponding linked list.
3. Traverse the linked list until we find **`stud47`**.



Chaining

- If we reach the end of the linked list and don’t find the desired student, then we know that student is *not contained in the hash table.*
  - There is *no other slot* at which the `Student` object would be stored.



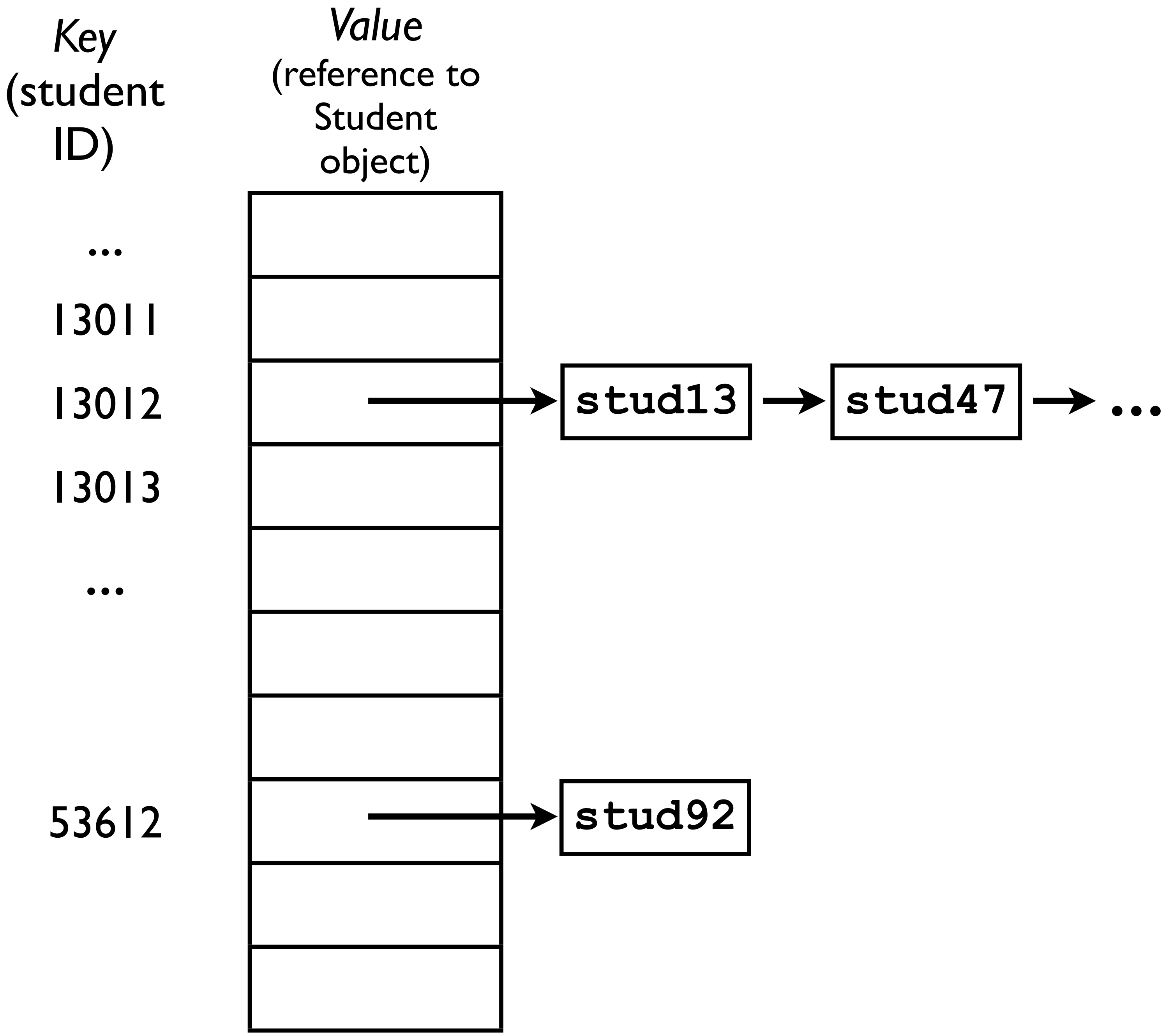
Chaining

- If we wish to *remove* a student (e.g., `stud13`), we just:

  1. Hash the student ID of the student we want to remove.

  2. Find that student in the linked list.

  3. "Splice it out".



Chaining: Performance analysis

- How fast are the **`add`**/**`remove`**/**`find`** operations for a hash table with chaining?
- To find an object in a hash table, we must:
  - Hash the key. *O*(1)
  - Jump to that array index. *O*(1)
  - Traverse through the linked list at that index. *O*(?)

Chaining: Performance analysis

- In the *worst case*, *all N* objects stored in the hash table will hash to the *same array index*.
  - This means that the linked list at that index will be *N* elements long.
    - To find an arbitrary element in a linked list we need *O*(*n*) time.
    - This is no better than just using a linked list by itself!

Chaining: Performance analysis

- However, in the *average* case, a hash table performs much better:
  - Given *M* slots and *N* actual objects stored, the average list length for any array slot is *N*/*M*.
  - Then, the average time to access any arbitrary object is *O*(1 + N/M).
  - Now, suppose that we always make sure that *M* > *N*.
    - Then *N*/*M* < 1.
    - Hence, average-case time cost is O(1+*N*/*M*)=O(1).

Open addressing

- An alternative to *chaining* is *open addressing*.
- With *open addressing*, there are no linked lists associated with array slots.
  - Instead, if a given slot is already "full", then the hash table "tries another one".
  - There are different strategies for "finding another one".
- Confusingly, open addressing is sometimes also known as *closed hashing*:
  - The "closed" refers to the fact that all data are stored within the array itself, not in a separate chain.

Open addressing

- There are different strategies for "finding another slot":
  - Simplest -- **linear probing**:
    - If `hashFunction(key)` maps into an index *i* that is already occupied, then try *i*+1.
    - If that doesn't work, try *i*+2, *i*+3, ..., etc.
    - If we get to *M*-1, we want to "wrap around" back to 0.
    - The index of the *j*th probe (where *j* starts at 0) is given by the expression:

      $(i+j) \% M$

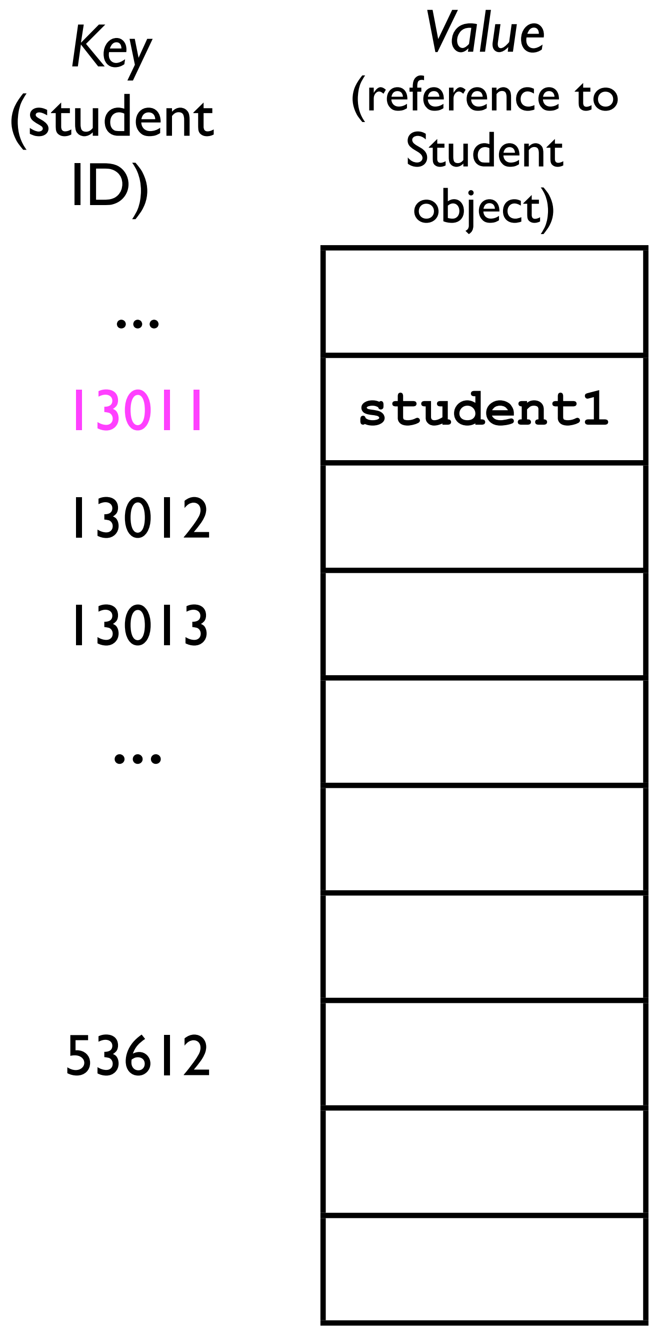
Open addressing

- Linear probing is simple to implement. However, it will tend to cause large amounts of *clustering* (more later).
- Better performance is achieved by **quadratic probing**:
  - If **`hashFunction(key)`** maps into an index *i* , then the index of the *j*th probe is given by the expression:

    $(i + c_0 * j + c_1 * j^2)$ % M

Linear probing

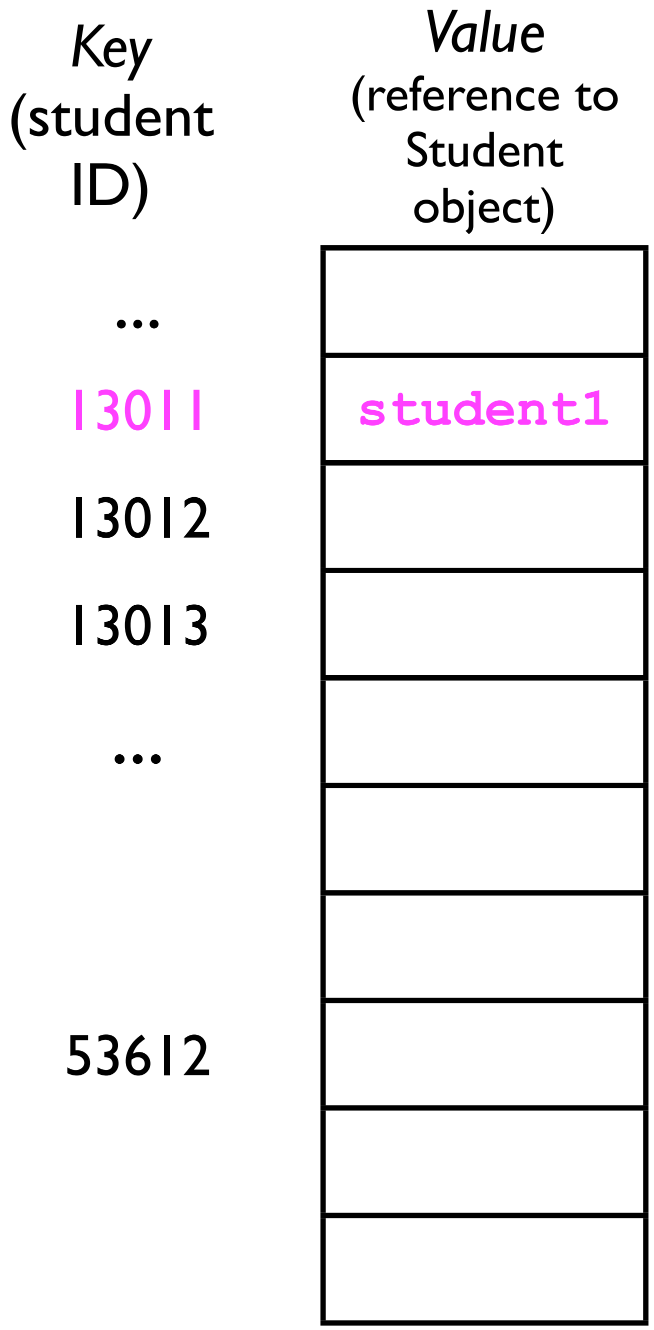
- Suppose our hash table contains M=100000 entries, and we wish to add `student2` to the hash table, and `hashFunction(studentID2) returns` 13011.
  - This slot is occupied by `student1`.
  - The next slot we try is (13011+1)%100000 = 13012.
  - Since 13012 is available, we can insert `student2` at that slot.



Key (student ID)	*Value* (reference to Student object)
...	
13011	`student1`
13012	
13013	
...	
53612	

Linear probing

- Suppose our hash table contains M=100000 entries, and we wish to add `student2` to the hash table, and `hashFunction(studentID2)` returns 13011.
  - This slot is occupied by `student1`.
  - The next slot we try is (13011+1) % 100000 = 13012.
  - Since 13012 is available, we can insert `student2` at that slot.



Key (student ID)

Value (reference to Student object)

...

13011 student1

13012

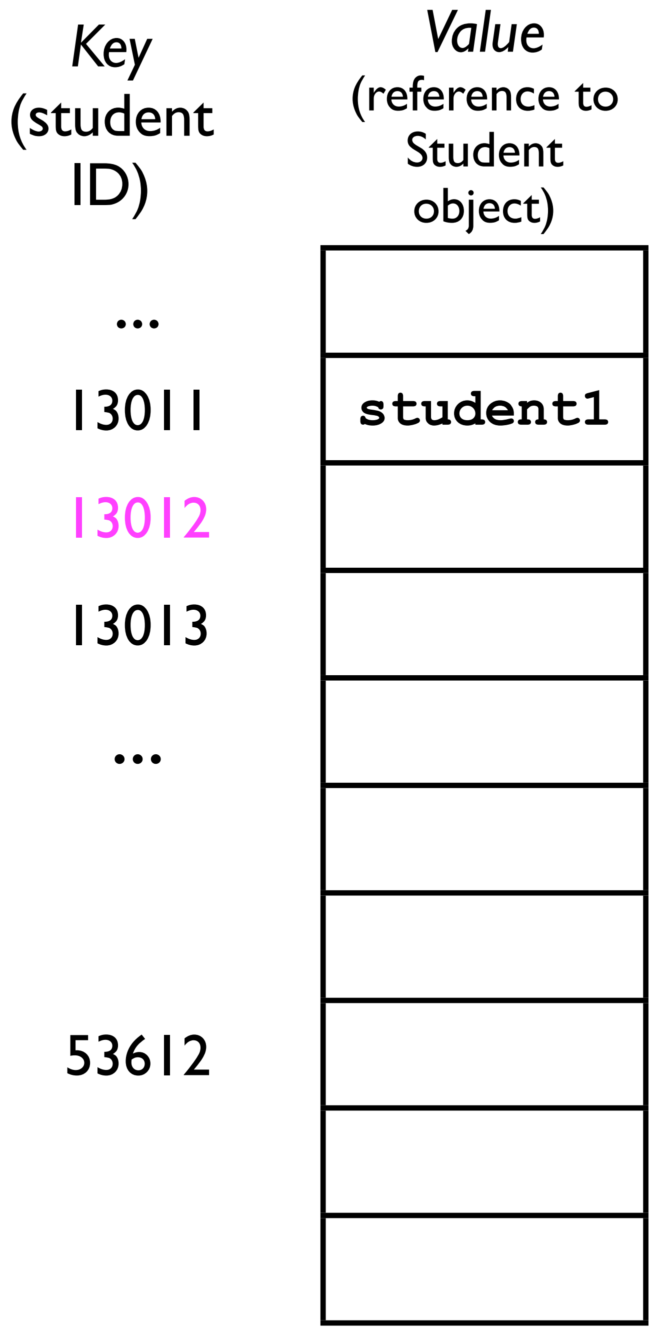
13013

...

53612

Linear probing

- Suppose our hash table contains M=100000 entries, and we wish to add `student2` to the hash table, and `hashFunction(studentID2)` returns 13011.
  - This slot is occupied by `student1`.
  - The next slot we try is (13011+1) % 100000 = 13012.
  - Since 13012 is available, we can insert `student2` at that slot.



Key (student ID)

Value (reference to Student object)

...

13011 `student1`

13012

13013

...

53612

Linear probing

- Suppose our hash table contains M=100000 entries, and we wish to add **`student2`** to the hash table, and **`hashFunction(studentID2)`** returns 13011.
  - This slot is occupied by **`student1`**.
  - The next slot we try is (13011+1) % 100000 = 13012.
  - Since 13012 is available, we can insert **`student2`** at that slot.

Key (student ID)	*Value* (reference to Student object)
...	
13011	**`student1`**
13012	**`student2`**
13013	
...	
53612	

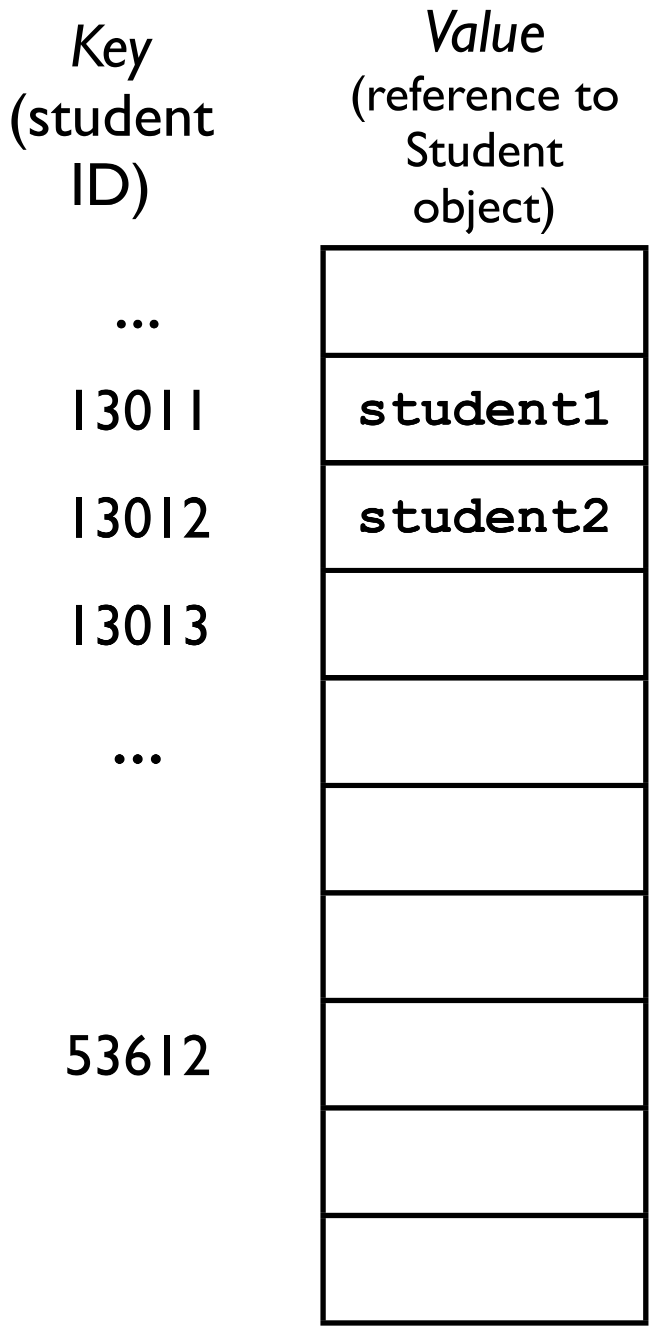
Linear probing

- When we later wish to find **`student2`**, we:

1. Compute its index using the hash function (*i*=13011).

2. Search down the array (using linear probing) until we find the correct object, *or* until we find an empty slot.

    - If we find an empty slot, then we know **`student2`** is *not* contained in the hash table.



Linear probing

- When we later wish to find `student2`, we:

  1. Compute its index using the hash function (*i*=13011).

  2. Search down the array (using linear probing) until we find the correct object, *or* until we find an empty slot.

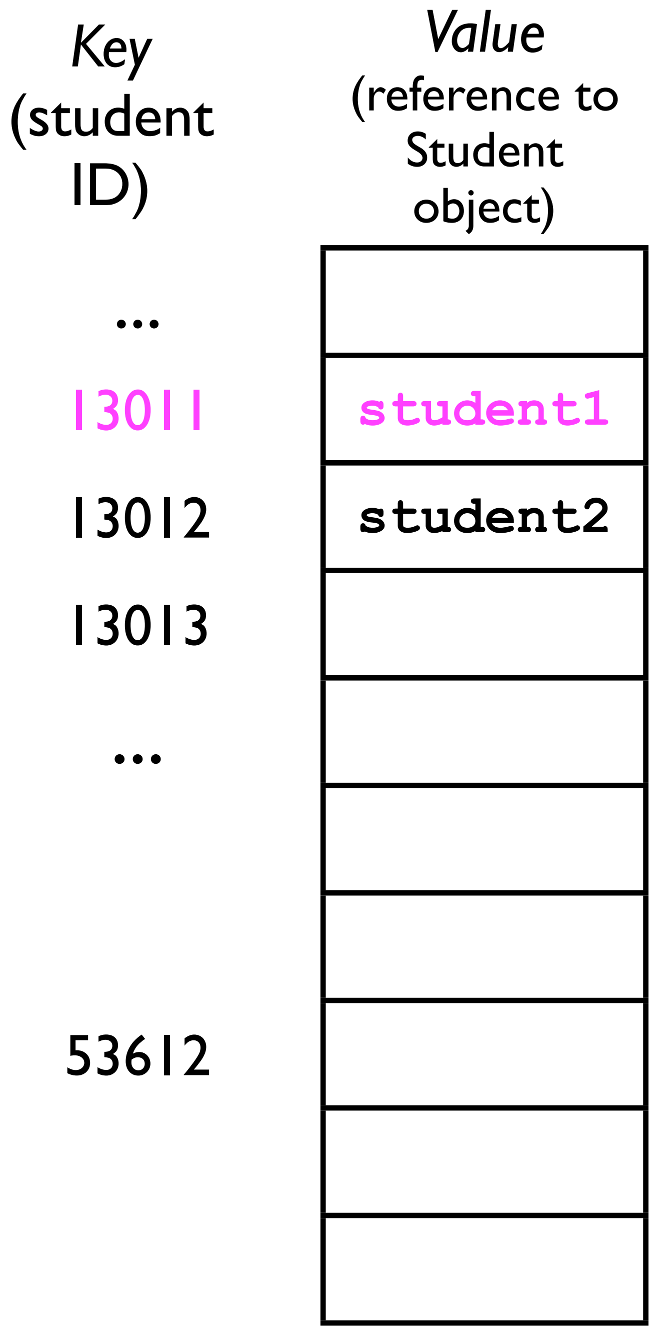
     - If we find an empty slot, then we know `student2` is *not* contained in the hash table.

Key (student ID)	*Value* (reference to Student object)
...	
13011	`student1`
13012	`student2`
13013	
...	
53612	

Linear probing

- When we later wish to find **`student2`**, we:

1. Compute its index using the hash function (*i*=13011).
2. Search down the array (using linear probing) until we find the correct object, *or* until we find an empty slot.
    - If we find an empty slot, then we know **`student2`** is *not* contained in the hash table.



Key (student ID)	*Value* (reference to Student object)
...	
13011	**`student1`**
13012	**`student2`**
13013	
...	
53612	

Linear probing

- When we later wish to find **`student2`**, we:

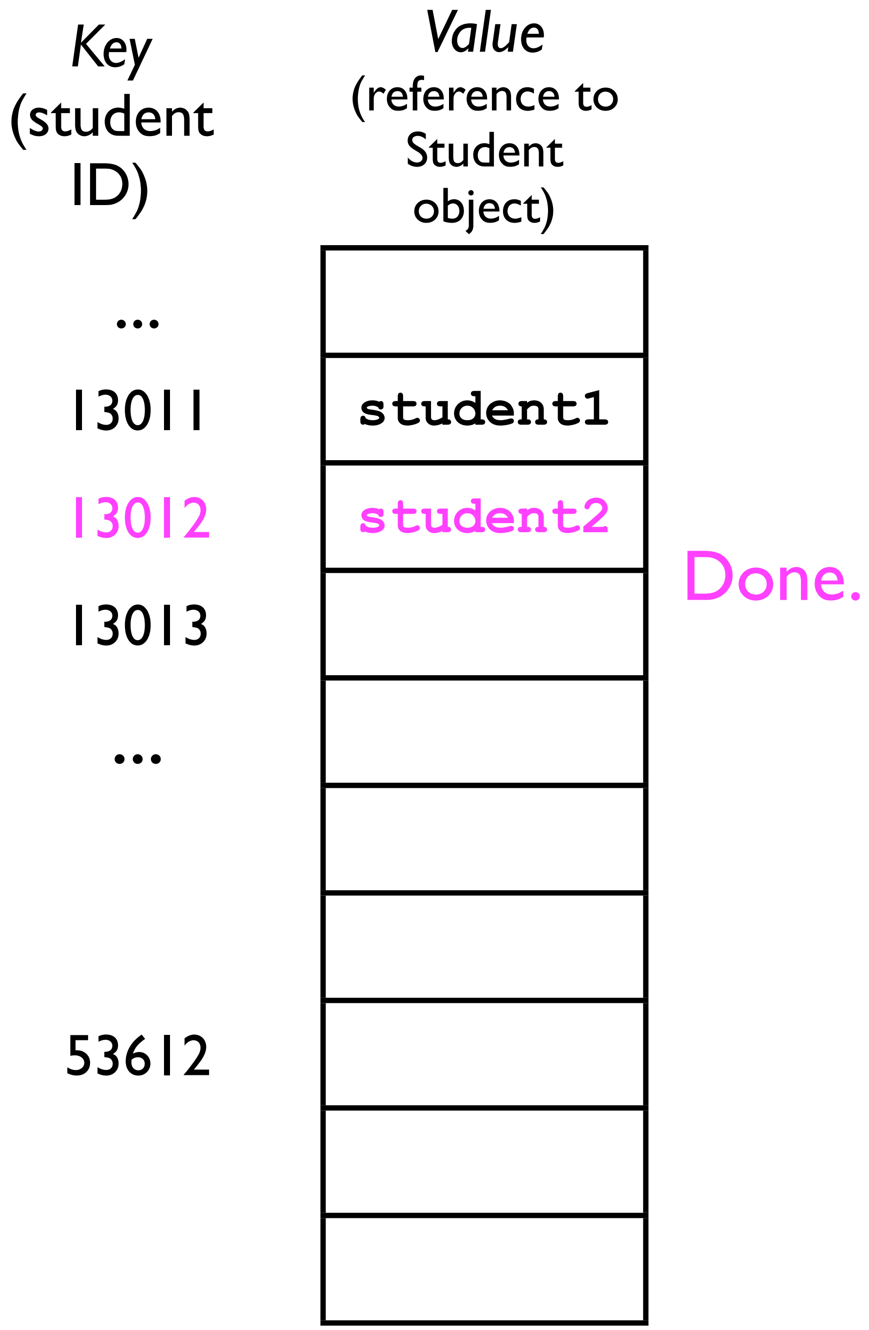
  1. Compute its index using the hash function (*i*=13011).
  2. Search down the array (using linear probing) until we find the correct object, *or* until we find an empty slot.
     - If we find an empty slot, then we know **`student2`** is *not* contained in the hash table.

Key (student ID)	*Value* (reference to Student object)
...	
13011	**`student1`**
13012	**`student2`**
13013	
...	
53612	

Linear probing

- When we later wish to find **`student2`**, we:

1. Compute its index using the hash function (*i*=13011).
2. Search down the array (using linear probing) until we find the correct object, *or* until we find an empty slot.
   - If we find an empty slot, then we know **`student2`** is *not* contained in the hash table.



Key (student ID)	*Value* (reference to Student object)	
...		
13011	`student1`	
13012	`student2`	Done.
13013		
...		
53612		

Linear probing

- Suppose we search for `student9`, whose hash code happens to also be 13011:

1. Compute its index using the hash function (*i*=13011).
2. Search down the array (using linear probing) until we find the correct object, *or* until we find an empty slot.
    - If we find an empty slot, then we know `student2` is *not* contained in the hash table.

Key (student ID)	*Value* (reference to Student object)
...	
13011	`student1`
13012	`student2`
13013	
...	
53612	

Linear probing

- Suppose we search for **`student9`**, whose hash code happens to also be 13011:

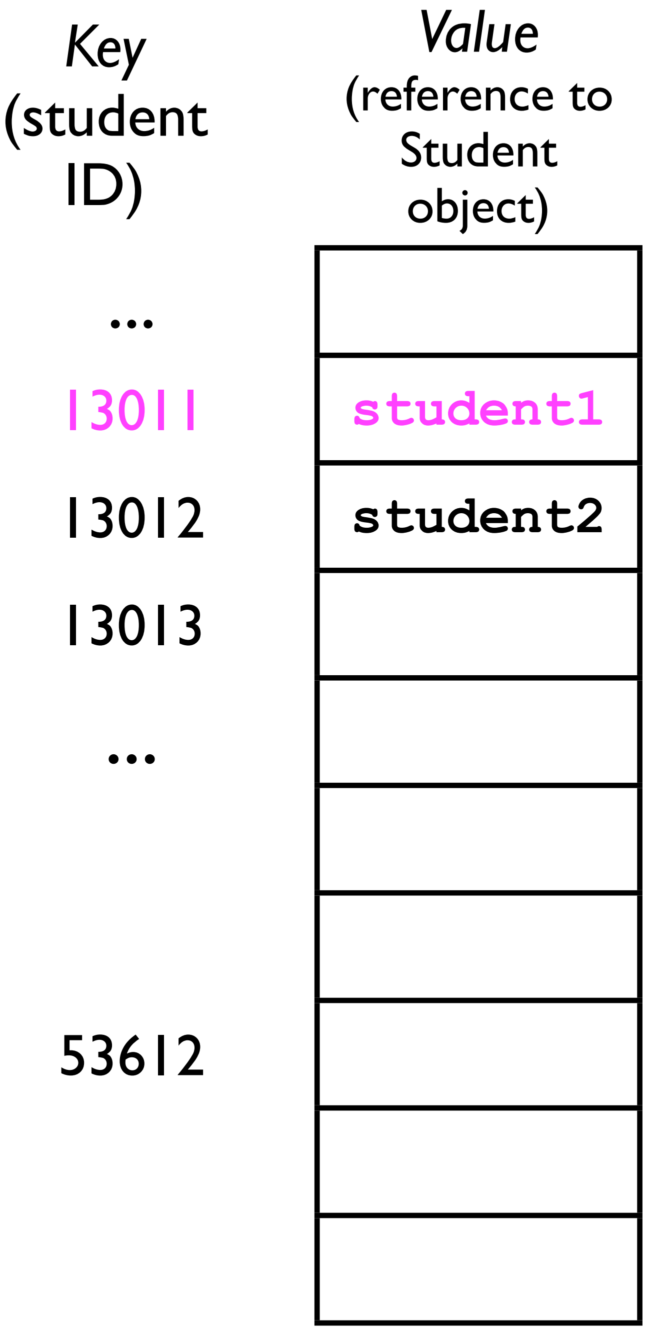
  1. Compute its index using the hash function (*i*=13011).
  2. Search down the array (using linear probing) until we find the correct object, *or* until we find an empty slot.
     - If we find an empty slot, then we know **`student2`** is *not* contained in the hash table.

Key (student ID)	*Value* (reference to Student object)
...	
13011	**`student1`**
13012	**`student2`**
13013	
...	
53612	

Linear probing

- Suppose we search for **`student9`**, whose hash code happens to also be 13011:

  1. Compute its index using the hash function (*i*=13011).
  2. Search down the array (using linear probing) until we find the correct object, *or* until we find an empty slot.
     - If we find an empty slot, then we know **`student2`** is *not* contained in the hash table.

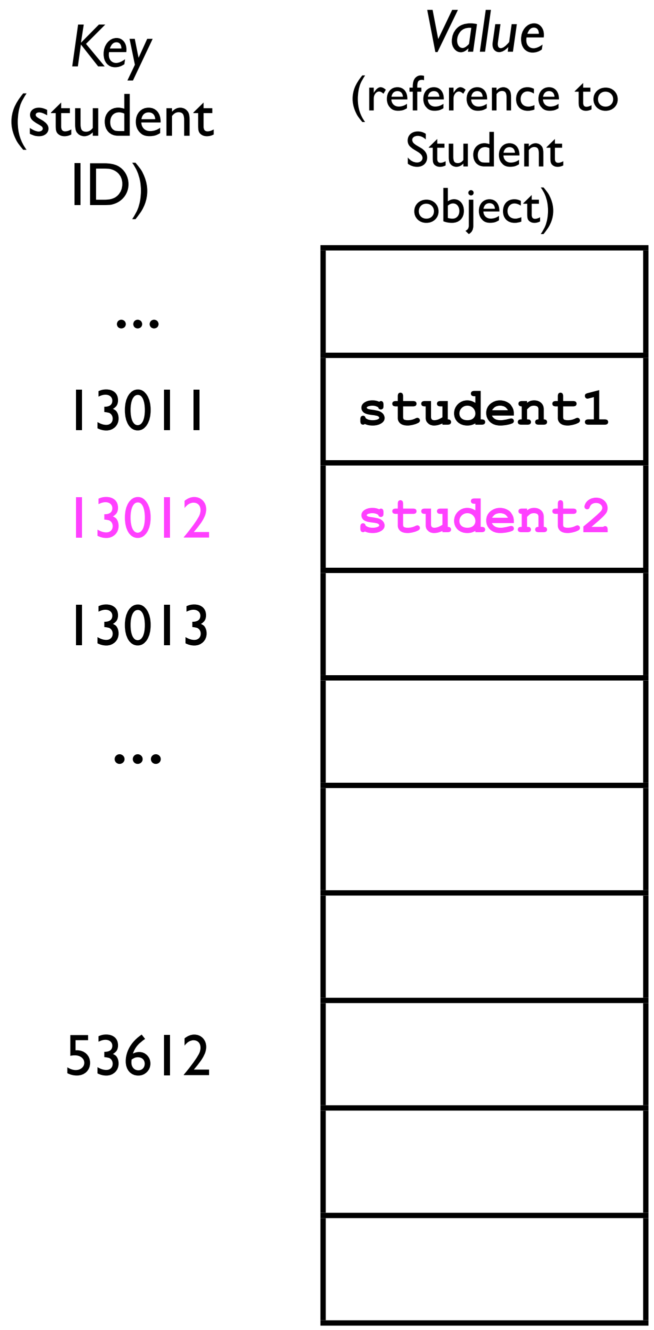


Key (student ID)	*Value* (reference to Student object)
...	
13011	`student1`
13012	`student2`
13013	
...	
53612	

Linear probing

- Suppose we search for **`student9`**, whose hash code happens to also be 13011:

1. Compute its index using the hash function (*i*=13011).
2. Search down the array (using linear probing) until we find the correct object, *or* until we find an empty slot.
   - If we find an empty slot, then we know **`student2`** is *not* contained in the hash table.



Key (student ID)	*Value* (reference to Student object)
...	
13011	**`student1`**
13012	**`student2`**
13013	
...	
53612	

Linear probing

- Suppose we search for **`student9`**, whose hash code happens to also be 13011:

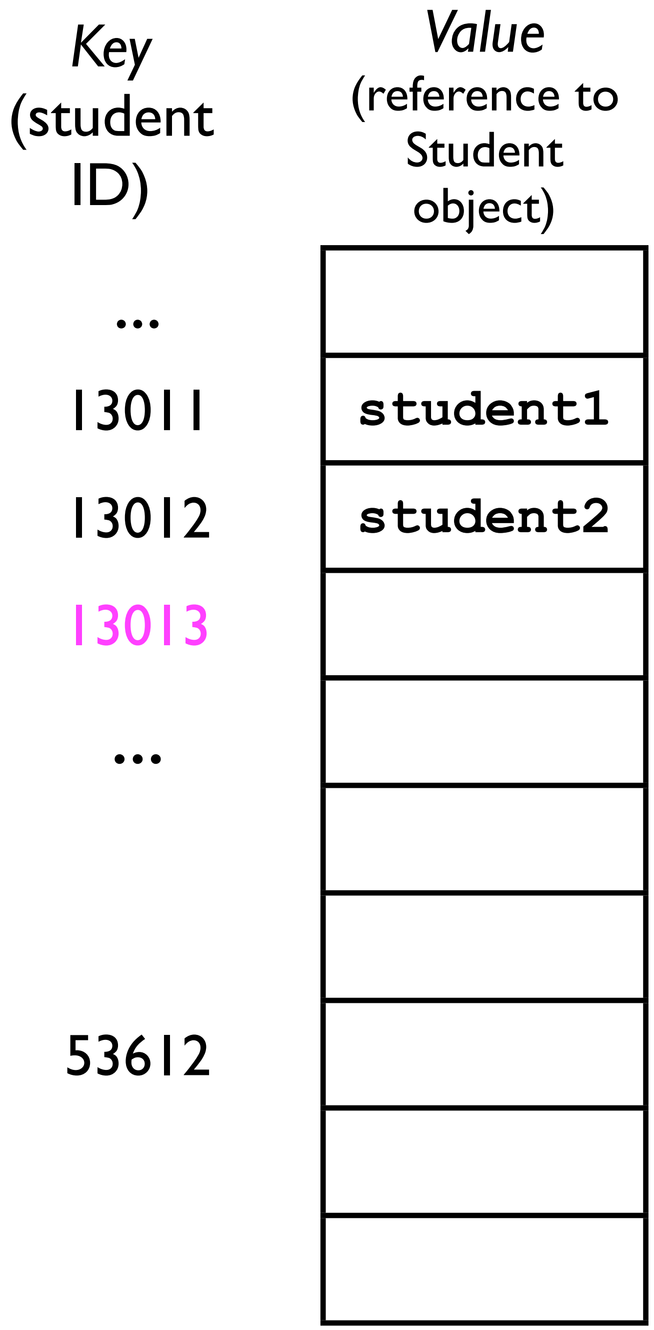
1. Compute its index using the hash function (*i*=13011).
2. Search down the array (using linear probing) until we find the correct object, *or* until we find an empty slot.
   - If we find an empty slot, then we know **`student2`** is *not* contained in the hash table.

Key (student ID)	*Value* (reference to Student object)
...	
13011	**`student1`**
13012	**`student2`**
13013	
...	
53612	

Linear probing

- Suppose we search for **`student9`**, whose hash code happens to also be 13011:

1. Compute its index using the hash function (*i*=13011).
2. Search down the array (using linear probing) until we find the correct object, *or* until we find an empty slot.
   - If we find an empty slot, then we know **`student2`** is *not* contained in the hash table.

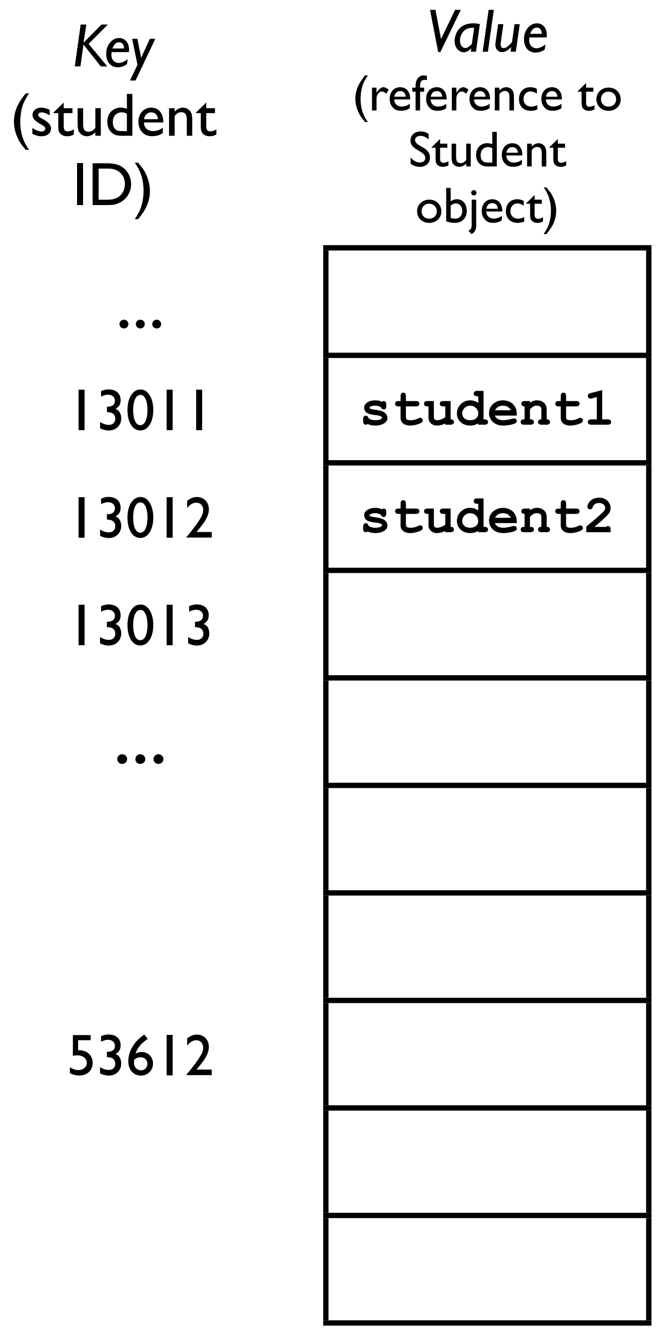


Key (student ID)	*Value* (reference to Student object)
...	
13011	**`student1`**
13012	**`student2`**
13013	
...	
53612	

Open addressing

- Open addressing requires less memory than chaining because there are no linked lists.
- However, they suffer from a few complications:

1. Removing an element.
2. Clustering.



Key (student ID)

Value (reference to Student object)

...

13011 `student1`

13012 `student2`

13013

...

53612

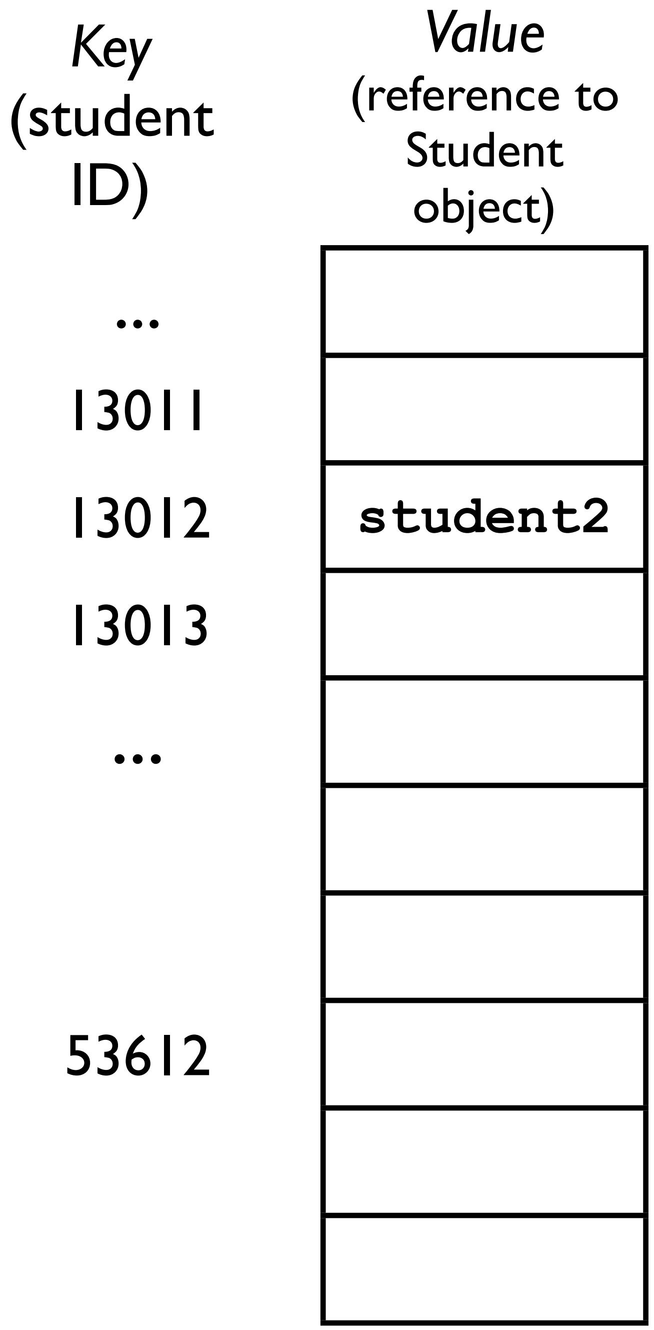
Removing an element

- Suppose we remove **`student1`** from the hash table.

Key (student ID)	*Value* (reference to Student object)
...	
13011	**`student1`**
13012	**`student2`**
13013	
...	
53612	

Removing an element

- Suppose we remove `student1` from the hash table.
- If we later search for `student2`, we will still hash to 13011, but find that it is *empty*.
  - Does that mean `student2` is not contained in the hash table?
  - No -- but we have to *record* that somehow.



Key (student ID)	*Value* (reference to Student object)
...	
13011	
13012	`student2`
13013	
...	
53612	

Removing an element

- One method of recording that an element was deleted is a *bridge*, a special element that indicates “empty, but keep looking.”
- If we later add another element, say `student5` that hashes to 13011, then we can *replace* the bridge with a real `Student` object.

Key (student ID)	*Value* (reference to Student object)
…	
13011	(bridge)
13012	`student2`
13013	
…	
53612	

Removing an element

- One method of recording that an element was deleted is a *bridge*, a special element that indicates "empty, but keep looking."
- If we later add another element, say `student5` that hashes to 13011, then we can *replace* the bridge with a real `Student` object.

Key (student ID)	*Value* (reference to Student object)
...	
13011	`student5`
13012	`student2`
13013	
...	
53612	

Clustering

- The other downside of open addressing is **clustering**.
- If too many keys hash to the same index -- *or to nearby indices* -- then the linear probing may become expensive.
- Consider the hash table to the right:
  - 13011-13016 are already occupied.
  - If we want to add another student `student7` who also hashes to 13011, then we have to step through 7 elements.
  - The longer the cluster, the higher the time cost for add/find/remove.

Key (student ID)	*Value* (reference to Student object)
...	
13011	`student5`
13012	`student1`
13013	`student9`
13014	`student8`
13015	`student3`
13016	`student4`
13017	
...	

Clustering

- The other downside of open addressing is **clustering**.
- If too many keys hash to the same index -- *or to nearby indices* -- then the linear probing may become expensive.
- Consider the hash table to the right:
  - 13011-13016 are already occupied.
  - If we want to add another student `student7` who also hashes to 13011, then we have to step through 7 elements.
  - The longer the cluster, the higher the time cost for add/find/remove.

Key (student ID)	*Value* (reference to Student object)
...	
13011	`student5`
13012	`student1`
13013	`student9`
13014	`student8`
13015	`student3`
13016	`student4`
13017	`student7`
...	

Open addressing

- Like chaining, *open addressing* guarantees O(1) time cost for the add/find/remove operations *so long as* *M* > *N*.
- Due to the bridging complication, open addressing is most useful when elements will *never be deleted*.
- The memory saved by not using linked lists can be used to allocate more slots ==> higher *M*.
    - The larger *M* is, the fewer collisions will occur, and the better the average-case performance will be.

HashTable ADT

- As with all the other containers we've examined in this course (linked lists, array lists, trees, heaps), hash tables offer **`add`**, **`find`**, and **`remove`** methods.
- In contrast to the other ADTs, these methods all take *O*(1) time in the average case.
- This *excellent time cost* comes at the expense of *increased memory usage* (since *M* must be much bigger than *N* to minimize collisions).